In search of preference shock risks: Evidence from longevity risks and momentum profits

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1. Introduction

Time-preference discount rate affects agents’ intertemporal choice (see, e.g., Frederick et al., 2002, for a review) and their demand for assets with different durations.

Therefore, shocks to agents’ time-preference discount rates move asset prices (Campbell, 1986; Albuquerque et al., 2016). In this paper, we explore the cross-sectional asset pricing implications of time-preference shocks arising from longevity risk in the stock markets. Longevity risk is a natural source of time-preference shocks. Intuitively, we expect agents to become impatient when facing a negative longevity shock and vice versa (Becker and Mulligan, 1997). From the duration matching perspective, agents invest more in stocks with longer durations when there is an unexpected increase in longevity to minimize the rollover risk in the future, because longevity risk generates greater uncertainty about future consumption. In particular, for momentum portfolios, we find that past losers have lower dividend growth and longer equity durations than past winners. Therefore, agents invest more in past losers (winners) for an unexpected increase (decrease) in longevity. Because stocks with longer durations have lower expected returns (see, e.g., Dechow et al., 2004;
Lettau and Wachter, 2007; Da, 2009; Binsbergen and Koijen, 2017; Weber, 2018), overall, previous losers underperform. We show that longevity risk captures most momentum profits observed in the US and UK markets, and the pricing power of longevity risk comes from the short-run risk component, i.e., a business cycle component.

Longevity risk represents unexpected shocks to life expectancy. Overall, people lived longer than expected in the last century, largely due to economic development and improved healthcare. However, changes in life expectancy are quite volatile. There are negative shocks resulting from economic recessions, epidemics, natural disasters, wars, or social and political disturbances. For example, in the US, life expectancy increased from 74.31 years in 1963 to 81.69 years in 2014, with an annual average increase of 0.14 years and an annual standard deviation of 0.15 years. This is significant, as it implies that longevity increases by 3.36 hours per day. Take the year of 2014 as an example. The average age in the US is 38.58 years, and if we do not consider further longevity improvement the expected remaining lifespan is 42.11 years. However, if we consider the effect of longevity increase, there could be 48 remaining years, which is much longer than the 42.11 years expected in 2014. Such changes in investment horizon affect agents’ intertemporal consumption and investment decisions, which in turn affect asset prices.

Longevity risk can be interpreted as shocks to time preferences, and it affects the pricing kernel via two channels. First, longevity increases with the time-preference discount rate. This is a direct channel. For example, a positive longevity shock induces agents to plan for a longer horizon and to place more weight on future utilities because future consumption becomes more likely, which means that agents become more patient. As a result, agents consume less today and save more in long-term risky assets, which implies lower expected returns. Second, time-preference shocks affect the cross-sectional income inequality, while longevity risk may capture income inequality and hence reveal time-preference shocks. This is an indirect channel. For example, Krusell and Smith (1998); Suen (2014), and Hubmer et al. (2016) show that stochastic time-preference rates across individuals affect cross-sectional inequality. Chetty et al. (2016) find that higher inequality is associated with higher mortality, because unequal societies could hurt the health and longevity of individuals due to economic and social reasons (Pickett and Wilkinson, 2009). Since counterfactual income inequality is positively priced (Constantinides and Duffie, 1996; Johnson, 2012), longevity risk is negatively priced.

Following Albuquerque et al. (2016), we model longevity risk via a stochastic time-preference shock process in the recursive preferences setting. This implies a consumption-based three-factor model, which includes longevity risk (time-preference shocks), consumption growth rate, and the market portfolio. This can be reduced to a two-factor model, including only the longevity factor and consumption factor, if we consider a power utility specification. Empirically, we adopt two different measures of longevity risk. Our first measure is a model-free one, which is computed as the innovations of the weighted average period life expectancy. Our second measure is based on the mortality risk. We use the Poisson Lee-Carter (PLC) model (Brouhns et al., 2002) to estimate a mortality index. We measure mortality risk as the innovations in the mortality index. Hence, a positive mortality shock means a negative shock to longevity. These two measures are highly correlated, with a correlation coefficient of $-0.99$. We find that longevity risk is significantly priced in the cross section of various test assets, especially for the momentum portfolios. Fig. 1 depicts the annual longevity risk (see the measurement details in Section 3) and the momentum factor over 1963–2014 in the US markets. Longevity risk closely tracks the momentum factor, with a correlation coefficient of $-0.26$. Momentum profits are low when longevity risk is high, which is strikingly evident in the momentum crash in 2009. This suggests that longevity risk is an important source of momentum profits.

We proceed to test the pricing power of the longevity factor in two ways. First, we apply the two-step generalized method of moments (GMM) to directly estimate the consumption-based models. Our test assets include six size and book-to-market sorted portfolios, six size and investment sorted portfolios, six size and operating profitability sorted portfolios, and six size and momentum sorted portfolios. We find that longevity risk is significantly priced. A one-year increase in longevity corresponds to a decrease of 5.17% in asset returns. The consumption-based three-factor model prices the test assets well, with a small root-mean-square error of about 1% per year.

Second, we employ the standard time-series and cross-sectional asset pricing tests. We construct a mimicking consumption portfolio and a mimicking longevity portfolio to test the consumption-based models. The Fama–MacBeth regressions of 25 size and momentum sorted portfolios show that longevity risk has a negative price of $-0.83$% per month ($t$-statistic $=-6.50$). The time-series regressions show that prior winners have negative loadings on longevity risk, while prior losers have a positive exposure. As longevity risk is negatively priced, this explains the return differences between prior winners and losers. The consumption-based three-factor model and its two variations, i.e., the two-factor model and the Fama–French three-factor model augmented with the longevity

1 The economic consequences of longevity risk have been widely noted from the insurance, health, and economic growth perspectives (e.g., Murphy and Topel, 2006; Hall and Jones, 2007). For example, IMF (2012) estimates that each additional year of life expectancy adds $3\% - 4\%$ to the present value of the liabilities of a typical defined benefit pension, and a three-year increase of life expectancy would cost 5$\%$ of 2010 GDP in developed economies.

2 We thank Robert Dittmar (the referee) for suggesting this channel.

3 One might suggest that longevity risk can be transferred through annuities, medical insurance, or public plans for individuals. However, some nondiversifiable longevity risk remains at the aggregate level. Moreover, uninsurable labor income shocks could affect longevity and make it uninsurable as well.

4 Slightly abusing the notation, we refer to both measures as longevity risk but clearly indicate the exact measure when necessary.
factor, perform well for various test assets. We perform extensive robustness checks and find that the results are robust to different sample periods, alternative test assets, different data frequencies, different longevity measures, and both US and UK markets.

Our paper speaks directly to the momentum literature, started by Jegadeesh and Titman (1993). Momentum strategies have been confirmed in various markets and asset classes. Empirically, momentum seems related to liquidity risk (Pástor and Stambaugh, 2003), consumption risk (Bansal et al., 2005), credit ratings (Avramov et al., 2007), macroeconomic risks like market state and industrial production growth (Chordia and Shivakumar, 2002; Cooper and Hameed, 2004; Liu and Zhang, 2008), stock performances 12 to 7 months before portfolio formation (Novy-Marx, 2012; Goyal and Wahal, 2015), ranking period return difference between past winners and losers (Huang, 2015), and earnings momentum (Chordia and Shivakumar, 2006; Novy-Marx, 2015). Behavioral models like Barberis et al. (1998); Daniel et al. (1998), and Hong and Stein (1999) attribute momentum profits to underreaction or delayed overreaction to information. Many risk-based explanations have also been proposed, either theoretically or empirically. Johnson (2002) and Sagi and Seasholes (2007) argue that past winners are inherently riskier. Vayanos and Woolley (2013) propose fund flows as a driver of momentum and reversal. Liu and Zhang (2014) and Hou et al. (2015) show that an investment-based model can partially capture the momentum profits. Unfortunately, the existing behavioral and rational explanations of momentum are far from conclusive. For example, momentum strategies are highly volatile and experience infrequent but severe losses in panic states (Daniel and Moskowitz, 2016), which challenges existing rational and behavioral explanations.

We are keen to understand the possible aggregate risks behind momentum profits. To this end, this paper taps the literature on demographic changes and time-preference shocks. Specifically, we examine the cross-sectional asset pricing implications of time-preference shocks introduced by longevity risk. We dig deeply to understand the economic links between momentum profits and longevity risk, which we summarize below. First, we show that prior winners (losers) provide hedging against mortality (longevity) risk, because winners experience higher dividend growth and thus have much shorter equity durations than losers. From the duration matching perspective, when facing a negative shock in longevity (i.e., a positive shock in mortality), agents invest more in assets with shorter durations, e.g., the winners portfolio, to minimize the rollover risks in the future. Thus, winners (losers) have negative (positive) exposure to longevity risk. Since longevity risk is negatively priced, winners (losers) have higher (lower) future returns. Second, as longevity risk varies over time, agents’ preferences for longer or shorter duration stocks change over time, which leads to time-varying momentum profits. For example, momentum profits are low when longevity risk is high. Let us inspect the largest momentum crash, which is in 2009. Innovation in life expectancy is only 0.05 years in 2008 (the financial crisis year) but 0.33 years in 2009, coinciding with the huge loss of momentum strategy in 2009. In fact, as shown in Fig. 1, longevity risk closely follows the three largest momentum crashes over the sample period, e.g., in 1975, 2003, and 2009, because longevity risk is unusually high in these three years. The ability of longevity risk to capture the momentum crashes is notable and distinguishes it from most existing explanations of momentum profits. Third, the frequency domain analysis directly shows that longevity risk and the momentum factor share a common business cycle component with a period of 2.74 years, which is missing in the Fama–French three factors. This is consistent with the findings that momentum relates to business cycles (Chordia and Shivakumar, 2002; Cooper and Hameed, 2004). Following the

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5 See Footnote 1 of Daniel and Moskowitz (2016) for references therein.

6 Some strategies have been proposed to manage the risks of momentum investment (Barroso and Santa-Clara, 2015; Gulen and Petkova, 2015).
long-run risk literature, we further show that longevity risk is negatively associated with the short-run consumption risk, but it is positively related to the long-run consumption risk. Therefore, the explanatory power of the longevity factor indeed comes from the short-run risk component, i.e., the business cycle component. Fourth, we show that longevity risk may reveal income inequality. We find that longevity decreases with income inequality, and income inequality partially contributes to the longevity factor. Fifth, Jegadeesh and Titman (1993) argue that for a risk-based explanation of momentum, the risk factors must be positively serially correlated. We find that the annual pricing kernel constructed from the consumption-based model is indeed positively serially correlated. This further validates our risk-based explanations of momentum strategy.

This paper adds to the literature on time-preference shocks and asset pricing. For example, Campbell (1986) considers the impacts of taste shocks on asset returns under a power utility case. Our paper is closely related to Maurer (2012) and Albuquerque et al. (2016). Maurer (2012) considers shocks to time preferences in an endowment economy and finds that time-preference shocks drive the equity premium, while consumption growth is of little importance. Albuquerque et al. (2016) consider exogenous demand shocks arising from random changes in investors’ time preferences and use them to explain the weak correlation between asset prices and fundamental variables at the aggregate level. Unlike the existing literature, which often assumes an exogenous and theoretical process of taste shocks, our paper provides a direct measure of time-preference shocks from longevity risk and examines its pricing power over the cross-section of various test assets.

This paper also relates to the literature on income inequality and asset pricing. Constantinides and Duffie (1996); Johnson (2012); Zhang (2014), and Brogaard et al. (2015) show that countercyclical income dispersion can generate a high equity premium. Pickett and Wilkinson (2009) and Chetty et al. (2016) show that inequality could affect longevity. This paper adds to the literature by studying the cross-sectional implications of income inequality, revealed by longevity risk.

Our paper also belongs to the literature on the asset pricing implications of longevity risk. Maurer (2014, 2015) and Kojen et al. (2016a) consider the optimal portfolio choice problem with mortality risk. Bisetti et al. (2017) empirically show that longevity risk is an aggregate risk and investors with a long (short) horizon want to hold more (less) a longevity-linked security. Intuitively, we expect that demographic changes would lead to changes in consumer behavior, which would then affect asset prices. For example, Dellavigna and Pollet (2007) and Kojen et al. (2016b) show that the long-term demand changes driven by demographic shifts predict abnormal returns in some industries, like the healthcare sector. Our paper adds to the literature by studying the cross-sectional implications of longevity risk in the stock markets, instead of industry-specific effects.

The rest of the paper proceeds as follows. In Section 2, we explore the asset pricing implications of time-preference shocks arising from longevity risk. In Section 3, we discuss the empirical measure of longevity risk and GMM estimation of the model. Section 4 performs the cross-sectional asset pricing tests of the longevity factor. Section 5 provides extensive robustness checks. Section 6 further investigates the mechanism underlying the pricing of longevity risk. Finally, Section 7 concludes.

2. Longevity risk and asset pricing

To guide our empirical exercises, we motivate longevity risk as shocks to time preferences via two different channels in a heterogeneous agents setting. First, longevity is related to the income level, which influences the time-preference rates. This is a direct channel. Second, longevity risk can also capture the cross-sectional income dispersion in a population and hence time-preference rates indirectly. Both channels suggests that longevity risk can be observationally equivalent to shocks to time preferences. Then we generalize this idea to the recursive preferences specification used by Albuquerque et al. (2016) and demonstrate that longevity risk is a pricing factor in a consumption-based model.

2.1. Motivation: longevity risk and time-preference shocks

Consider an incomplete market with a continuum of heterogeneous agents of unit mass. Assume all agents have homogenous beliefs and are endowed with an identical power utility, e.g., \[ U(C_t) = C_t^{1-\gamma} \], where \( C_t \) is the consumption of agent \( i \) at time \( t \) and \( \gamma \) is the relative risk aversion. Assume agents have homogeneous time-preference discount rates, e.g., the subjective discount rate of the population at time \( t \) is \( \beta_t \). This setting is close to Constantinides and Duffie (1996), but we allow for time-varying time-preference rates. Hence, the pricing kernel of agent \( i \) can be written as

\[ M_t^i = \frac{\beta_{t+1}}{\beta_t} \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma}. \]  
(1)

Following Constantinides and Duffie (1996), we assume that the consumption growth rate of agent \( i, \frac{C_{t+1}}{C_t} \), satisfies

\[ \frac{C_{t+1}}{C_t} = \frac{C_{t+1}}{C_t} \epsilon_{t+1} \]  
(2)

where \( \frac{C_{t+1}}{C_t} \) is the aggregate consumption growth rate, and \( \epsilon_{t+1} \) denotes the idiosyncratic consumption growth. Assume that \( \log \epsilon_{t+1} \) is normally distributed with a mean of \( -\frac{1}{2} \sigma_{y,t+1}^2 \) and a variance of \( \sigma_{y,t+1}^2 \). \( \sigma_{y,t+1}^2 \) denotes the cross-sectional dispersion of consumption growth across individuals, which is income inequality if we assume that consumption equals income. Conditional on \( \sigma_{y,t+1}^2 \) and \( \frac{C_{t+1}}{C_t} \), assuming that \( \epsilon_{t+1} \) is independent across individuals, then

\[ \beta_0 \] is normalized as 1 and the one-period time-preference rate is \( \frac{\beta_{t+1}}{\beta_t} \).
the pricing kernel of any agent can be written as
\[ M_{t+1} = \beta_{t+1} \exp \left\{ \frac{\gamma (1 + \gamma)}{2} \sigma^2_{\gamma t+1} \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \right\}. \] (3)

Time-preference discount rates seem to vary with income level. For example, using the Panel Study of Income Dynamics (PSID) data, Lawrance (1991) finds that time-preference rates for poor families are three to five percentage points lower than those for rich families. That is, agents become more patient when their income increases. Assume that the aggregate income at time \( t \) is \( y_t \). Similar to Lawrance (1991), we parameterize the time-preference discount rate as a log-linear function of the aggregate income at time \( t \), as follows:
\[ \ln \beta_t = b_0 + b_1 \ln y_t, \] (4)
where \( b_0 \) and \( b_1 \) are constants with \( b_1 > 0 \). A positive \( b_1 \) means time-preference discount rate increases with income. Then the one-period time-preference discount rate for the population at time \( t \) is
\[ \frac{\beta_{t+1}}{\beta_t} = \left( \frac{y_{t+1}}{y_t} \right)^{b_1}. \] (5)

Previous studies suggest that income level is an important factor determining the population longevity. Longevity tends to increase with income (e.g., Bloom and Canning, 2000).\(^8\) Assume a simple parametric form of the mean life expectancy for the population \( (E_t) \) on aggregate income \( (y_t) \) at time \( t \), as follows:
\[ \ln E_t = e_0 + e_1 \ln y_t, \] (6)
where \( e_0 \) and \( e_1 \) are constants with \( e_1 > 0 \). A positive \( e_1 \) implies that life expectancy increases with income. Then the population life expectancy grows at a rate of
\[ \frac{E_{t+1}}{E_t} = \left( \frac{y_{t+1}}{y_t} \right)^{e_1}. \] (7)
Substituting Eq. (5) into Eq. (7), we see that longevity growth directly captures changes in the time-preference rates:
\[ \frac{\beta_{t+1}}{\beta_t} = \left( \frac{E_{t+1}}{E_t} \right)^{e_1}. \] (8)

Next, similar to Constantines and Duffie (1996), we assume that income inequality is negatively related to the aggregate income growth, as follows:
\[ \sigma^2_{\gamma t+1} = c_0 + c_1 \ln \left( \frac{y_{t+1}}{y_t} \right), \] (9)
where \( c_0 \) and \( c_1 \) are constants with \( c_1 < 0 \). Substituting Eq. (7) into this equation, we have
\[ \sigma^2_{\gamma t+1} = c_0 + \frac{c_1}{e_1} \ln \left( \frac{E_{t+1}}{E_t} \right). \] (10)
This implies that population longevity decreases with income inequality, as \( c_1 < 0 \) and \( e_1 > 0 \). This is consistent with findings in Rodgers (1979) and Chetty et al. (2016). Hence, longevity may reflect income inequality as well.

Substituting Eqs. (8) and (10) into Eq. (3) gives
\[ M_{t+1} = \exp \left\{ \frac{\gamma (1 + \gamma)}{2} c_0 \left( \frac{E_{t+1}}{E_t} \right)^{\frac{y_{t+1} + y_t}{y_t t+1}} \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \right\}. \] (11)

Define \( \beta \) as
\[ \beta = \exp \left\{ \frac{\gamma (1 + \gamma)}{2} c_0 \right\}, \] (12)
and \( L_t \) as
\[ L_t = \frac{E_{t+1}}{E_t} \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma}, \] (13)

\( L_t \) can be interpreted as the longevity factor at time \( t \). Then the pricing kernel can be rewritten as
\[ M_{t+1} = \beta^t_{L_{t+1}} \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma}, \] (14)
where longevity is an additional pricing factor. Alternatively, we can view the longevity factor as the stochastic time-preference rates.

From the asset pricing perspective, Eqs. (3), (8), and (10) say that longevity affects the pricing kernel through two channels. First, longevity influences the time-preference discount rate. As longevity increases, agents become more patient (see Eq. (8)). This implies a higher marginal utility in the future and hence a negative price of longevity risk. Second, longevity captures income inequality, as it negatively correlates with the income inequality \( \sigma^2_{\gamma t} \) (see Eq. (10)). Moreover, counter-cyclical income dispersion can contribute to the high equity premium (Constantines and Duffie, 1996; Johnson, 2012).\(^5\) Empirically, Johnson (2012) finds that assets hedging against inequality risk command lower returns in US markets.\(^10\) Zhang (2014) and Brogaard et al. (2015) also find that higher income inequality predicts a higher equity premium in various countries. Since income inequality is positively priced and negatively correlated with longevity, this channel also suggests a negative price of longevity risk. Overall, both channels imply that longevity risk is negatively priced.

In fact, both channels suggest that longevity factor captures time-preference shocks. This is clear for the time-preference rate channel shown in Eq. (8). This is also true for the income inequality channel. Kruessel and Smith (1998); Suen (2014), and Hubner et al. (2016) show that stochastic time-preference rates across individuals affect cross-sectional inequality. Since longevity risk captures income inequality, it reveals time-preference shocks to the population. In short, Eqs. (13) and (14) show that we can treat longevity factor observationally equivalent to shocks to time preferences.

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\(^8\) The exact causality is unclear. On one hand, higher income gives agents more resources that promote health, which increases the longevity. On the other hand, longevity enhances the productivity, education, and demographic dividends, which improve income of individuals.

\(^9\) Using the PSID data, Storesletten et al. (2004) confirm the counter-cyclicality of income growth dispersion. Using a very large dataset from the US Social Security Administration, Guvenen et al. (2014) also find counter-cyclical income growth dispersion among higher income groups.

\(^10\) Johnson (2012) considers the impacts of inequality in incomplete markets where agents' utilities are defined over relative consumption. Relative consumption captures agents' status concerns. Such status concerns could affect asset prices (e.g., habit models) and longevity (due to social and psychological factors).
2.2. Longevity risk and asset prices: a consumption-based model

In this section, we extend the idea motivated before to the more general Epstein–Zin recursive preferences, i.e., modeling longevity risk through the random time discount rate process. This setup is similar to that of Albuquerque et al. (2016) and Hubmer et al. (2016). We assume investors are endowed with recursive preferences as follows:

$$U_t = \left( L_t C_t^{\frac{1}{1-\gamma}} + \beta \left[ \mathbb{E}_t U_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}},$$  \hspace{1cm} (15)

where $L_t$ captures the random time discount due to longevity shocks, $C_t$ is the consumption at time $t$, $\beta$ is the usual time discount, $\gamma$ measures the relative risk aversion, $\theta = \frac{1}{1-\gamma}$, and $\psi$ is the elasticity of intertemporal substitution. When $\gamma = \frac{1}{\psi}$, the recursive preferences reduce to the power utility case.

Then the pricing kernel is

$$M_{t,t+1} = \beta \frac{L_{t+1}}{L_t} \left[ \frac{C_{t+1}}{C_t} \right]^{-\frac{1}{\psi}} \left[ \frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_t U_{t+1}^{1-\gamma}} \right]^{\frac{1}{1-\gamma}}.$$  \hspace{1cm} (16)

As in Epstein and Zin (1991), let the return on the aggregate wealth portfolio be $R_{w,t+1}$; then we can rewrite the above pricing kernel as

$$M_{t,t+1} = \left[ \beta \frac{L_{t+1}}{L_t} \right]^{\frac{1}{\psi}} \left[ \frac{C_{t+1}}{C_t} \right]^{-\frac{1}{\psi}} R_{w,t+1}^{\psi}.$$  \hspace{1cm} (17)

The pricing kernel includes the usual consumption growth rate, the return on the aggregate wealth portfolio, and the changes in the time discount factor arising from longevity risk. Thus, longevity risk matters for asset pricing. Taking the logarithm of the pricing kernel, we have

$$m_{t+1} = \theta \log \beta + \theta \Delta l_{t+1} - \frac{\theta}{\psi} \Delta C_{t+1} + \left( \theta - 1 \right) r_{w,t+1},$$  \hspace{1cm} (18)

where $m_{t+1} = \log M_{t,t+1}$, $\Delta l_{t+1} = \log l_{t+1} - \log l_t$, $\Delta C_{t+1} = \log C_{t+1} - \log C_t$, and $r_{w,t+1} = \log R_{w,t+1}$. Therefore, the innovations of the pricing kernel can be written as

$$m_{t+1} - \mathbb{E}_t [m_{t+1}] = \theta \left( \Delta l_{t+1} - \mathbb{E}_t [\Delta l_{t+1}] \right)$$
$$- \frac{\theta}{\psi} \left( \Delta C_{t+1} - \mathbb{E}_t [\Delta C_{t+1}] \right)$$
$$+ \left( \theta - 1 \right) \left( r_{w,t+1} - \mathbb{E}_t [r_{w,t+1}] \right).$$  \hspace{1cm} (19)

This suggests a linear three-factor model. Define the factor vector as

$$f_{t+1} = \begin{pmatrix} \Delta l_{t+1} \\ \Delta C_{t+1} \\ r_{w,t+1} \end{pmatrix},$$  \hspace{1cm} (20)

and let the coefficient vector be

$$b = \begin{pmatrix} b_t \\ b_C \\ b_m \end{pmatrix} = \begin{pmatrix} -\frac{\theta}{\psi} \\ \frac{\theta}{1-\theta} \end{pmatrix};$$  \hspace{1cm} (21)

then by log-linearization we have

$$\frac{M_{t+1}}{\mathbb{E}[M_{t+1}]} \approx 1 + m_{t+1} - \mathbb{E}[m_{t+1}] = 1 - b' (f_{t+1} - \mu_f),$$  \hspace{1cm} (22)

where $\mu_f = \mathbb{E}[f_{t+1}]$, which are the unconditional means of the factors. From the basic asset pricing equation, we have

$$\mathbb{E}[R_{f,t+1}^e M_{t,t+1}] = 0 = \mathbb{E}\left[ \frac{M_{t,t+1}}{\mathbb{E}[M_{t,t+1}]} R_{f,t+1}^e \right].$$  \hspace{1cm} (23)

where $R_{f,t+1}^e$ is the excess return of asset $i$ at time $t+1$. This implies

$$\mathbb{E}[R_{f,t+1}^e (1 - b' (f_{t+1} - \mu_f))] = 0.$$  \hspace{1cm} (24)

The three-factor linear asset pricing model is

$$\mathbb{E}[R_{f,t+1}^e] = b_i \mathbb{Cov}(f_{t+1}, R_{f,t+1}^e) + \lambda \Sigma_{f}^{-1} \mathbb{Cov}(f_{t+1}, R_{f,t+1}^{e})$$
$$+ \mu_f \mathbb{E}[R_{f,t+1}^e].$$  \hspace{1cm} (25)

where $\Sigma_f$ is the covariance matrix of factors $f$; $\lambda$ contains the prices of factor risks and $\lambda = \Sigma_f b$.

In the special case of a power utility specification, we have a two-factor model:

$$\mathbb{E}[R_{f,t+1}^e] = b_i \mathbb{Cov}(\Delta l_{t+1}, R_{f,t+1}^e) + \mathbb{Cov}(\Delta C_{t+1}, R_{f,t+1}^e).$$  \hspace{1cm} (26)

3. Estimating the longevity risk

3.1. Data description

The annual US data of population, exposure, and mortality rates are obtained from the Human Mortality Database (HMD) (see Internet Appendix A for more details about data construction). Due to its accessibility, reliability, and consistency over time, it has been widely used in demographic and actuarial research. HMD uses the official population estimates available from the Census Bureau with several adjustments. For example, HMD adjusts the population estimates to exclude the military population to make the population estimates consistent over time. HMD also distributes the population in the open age interval into a single year of age and reestimates the population as of January 1 for each year. HMD takes the death data from the National Vital Statistics Reports by the National Center for Health Statistics. HMD distributes the death counts into a single year of age using cubic spline and then assigns deaths of unknown ages into each age category. For the purpose of our analysis, we use the mortality series from 1963 to 2014. The age range we consider is 0–99, as the number of deaths for age 100 and beyond is very volatile and the data are not as reliable.\footnote{Given the small exposure at such ages, their impacts on the longevity estimates are negligible.}

3.2. Estimating longevity shocks

We first measure longevity shocks with a model-free approach, which is computed as the first-order difference of the weighted average period life expectancy, $E_l$. Life expectancy at birth is a common measure of human longevity. However, it ignores the life expectancy, or decision horizon, of older members in an economy. To capture
this, we compute $E_t$ as life expectancy weighted by exposure across all ages, as follows:

$$E_t = \frac{\sum_{x=0}^{99} (x + \epsilon_{x,t}) \cdot \Delta E_{x,t}}{\sum_{x=0}^{99} \epsilon_{x,t}},$$

where $\epsilon_{x,t}$ is the period life expectancy for a person aged $x$ in year $t$, and $\Delta E_{x,t}$ is the corresponding exposure. Then longevity shocks can be computed as innovations in the life expectancy:

$$dE_t = E_t - E_{t-1}. \quad (27)$$

A positive shock to life expectancy implies an increase in longevity.

Longevity shocks are positive on average due to the increase in life expectancy in the last century. Longevity shocks are also very volatile, as shown in Fig. 1. Panel A of Table 1 provides descriptive statistics of longevity risk. $dE_t$, $\Delta E_t$ has a mean of 0.14 years, a standard deviation of 0.15 years, and a negative first-order autocorrelation of $-0.10$. Augmented Dickey-Fuller tests show that $E_t$ has a unit root, but longevity shocks ($dE_t$) are stationary.

### 3.3. GMM estimation

In this section, we directly test the pricing power of the consumption-based model over various assets via GMM estimation. The unconditional asset pricing Eq. (24) gives us the moment conditions. Following Cochrane (2005), we include the pricing factors as additional moment constraints. That is, we use the following moment conditions,

$$E\left[\frac{E_{x,t+1}}{E_{t+1}} \left(1 - b'(f_{t+1} - \mu_f) \right) \right] = 0, \quad (28)$$

where the coefficients $b$ and the factor means $\mu_f$ are to be estimated. We use a two-step GMM estimation with a Newey–West one-lag adjustment.

Our test assets include six size and book-to-market sorted portfolios, six size and investment sorted portfolios, six size and operating profitability sorted portfolios,

$$b_{x,t} = \frac{\sum_{x=0}^{99} \epsilon_{x,t} \cdot (x + \Delta E_{x,t})}{\sum_{x=0}^{99} \epsilon_{x,t}}.$$

where $\epsilon_{x,t}$ is the population estimate for age $x$ on January 1 in year $t$, and $\Delta E_{x,t}$ is the number of deaths for age $x$ in year $t$. The superscript $l (U)$ denotes the number of lower triangle (upper triangle) deaths in the Lexis diagram.

### 13

Alternatively, Eqs. (14) and (16) suggest that we can measure longevity shocks as the longevity growth rates. We find that these two measures provide similar results, as shown in Internet Appendix B. But measuring longevity risk as the first-order difference provides an easy way to interpret the GMM estimation results in the next section, e.g., the price of a one-year increase in longevity.

### 14

GMM estimates are often sensitive to the specifications, especially the weighting matrix. We evaluate various GMM estimation procedures in Internet Appendix B. To be conservative, we report two-step GMM with one-lag Newey–West adjustment as our main results.

and six size and momentum sorted portfolios. Consumption refers to the nondurable goods and services consumption. Kroencke (2017) argues that the national income and product accounts (NIPA) data have significant measurement errors due to time aggregation and the filters used. Thus, we follow his recommendation and use his unfiltered consumption data. Similar to Epstein and Zin (1991), we proxy the aggregate wealth portfolio using the stock market portfolio. For easy interpretation, we proxy the shocks to time preferences ($\Delta b_{x,t+1}$) by the longevity shocks ($\Delta E_t$). We obtain the annual data of test assets and the Fama–French three factors from Kenneth French’s website. Stock returns are adjusted by the Consumer Price Index and converted into real returns when necessary. As portfolios on investment and operating profitability start in 1964, our annual data are from 1964 to 2014.

Table 2 presents the GMM estimation results. We first consider the simple power utility case in Column (1). Then we move to the general recursive preferences in Column (2). Note that some coefficients are constrained in some cases, as shown in Eq. (21). For example, in the power utility case, $b_1$ must be $-1$, because $\gamma = \frac{1}{\psi}$ and $\theta = 1$. Also, in the recursive preferences case, $b_2 = 1 + b_1$. Therefore, we impose these restrictions during GMM estimation. For comparison, we also consider CAPM in Column (3) and the Fama–French three-factor model in Column (4). Panel A presents the coefficient estimates for $b$. Then, Panel B computes the implied price of risk for each factor ($\lambda$), i.e., $\lambda = \Sigma b$. Panel C reports the relative risk aversion ($\gamma$) and elasticity of intertemporal substitution ($\psi$) implied by the estimates in Panel A. That is, from Eq. (21), we know that $\gamma = 1 + (b_1 + b_2)$ and $\psi = -b_1/b_2$. Lastly, Panel D tests the goodness of fit for each model. We compute the adjusted $R^2$, root-mean-square errors (RMSE), and the Hansen’s J-test of overidentification. $R^2$ is defined as one minus the ratio of the cross-sectional variance of the pricing errors to the cross-sectional variance of realized average portfolio returns, following Campbell and Vuolteenaho (2004).

We first investigate our main model, the case of recursive preferences in Column (2). Shocks to the time preferences, i.e., longevity risk, are significantly priced. Column (2) shows that longevity risk has a coefficient of $-1.30$ with a $t$-statistic of $-5.30$, which corresponds to a price of risk of $-5.17$. That is, a one-year increase in longevity corresponds to a decrease of 5.17% in asset returns. This is close to the IMF (2012) estimate that each additional year of life expectancy increases 3%–4% to the present value of liabilities of a typical defined benefit pension plan. Consumption is significantly priced in the cross-section as well, with a coefficient of 25.42 ($t$-statistic=11.51), and the market factor has a price of risk of 2.61%. The main model implies a relative risk aversion of 25.13 and a small elasticity of

Admittedly, this proxy neglects many important assets in the market (e.g., real estate) as the stock market portfolio is only a subset of the aggregate wealth portfolio. However, we can interpret the stock market portfolio as the instrumental variable for the aggregate wealth portfolio in the GMM estimation, since the stock market portfolio should be highly correlated with the aggregate wealth portfolio.

Hall and Jones (2007) also use the reciprocal of life expectancy to measure the time discount rate.
Table 1
Longevity risk: descriptive statistics and relations with other factors.
Panel A summarizes the annual statistics of longevity risk (dE), the mimicking portfolio returns of longevity risk (PL, in %), and the mimicking portfolio returns of consumption risk (PC, in %). The full-sample data are used in estimating the mimicking portfolio returns. AR(1) denotes the first-order autocorrelation of each series. Panel B reports the sample means, standard deviation, Sharpe ratio, and correlations for the Fama–French three factors, momentum factor (MOM), the mimicking portfolio for longevity risk (PL), and the mimicking portfolio for consumption risk (PC), using annual data. Panel C reports the time-series regressions of the longevity factor against the Fama–French three-factor model or the Fama–French three-factor model augmented with the momentum factor, using monthly data. Panel D reports the time-series regressions of the consumption factor against the Fama–French three-factor model or the Fama–French three-factor model augmented with the momentum factor, using monthly data. The Newey–West t-statistics with six lags are in parentheses. The sample data are from 1963 to 2014.

Panel A: Descriptive statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Std. dev.</th>
<th>Min.</th>
<th>Max.</th>
<th>AR(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>dE</td>
<td>0.14</td>
<td>0.14</td>
<td>0.15</td>
<td>−0.26</td>
<td>0.47</td>
<td>−0.10</td>
</tr>
<tr>
<td>PL(%)</td>
<td>−9.81</td>
<td>−9.81</td>
<td>11.88</td>
<td>−30.56</td>
<td>40.70</td>
<td>0.04</td>
</tr>
<tr>
<td>PC(%)</td>
<td>13.15</td>
<td>15.18</td>
<td>14.22</td>
<td>−17.50</td>
<td>45.72</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Panel B: Factor means, volatilities, and correlations

<table>
<thead>
<tr>
<th>Factor</th>
<th>Mean (%)</th>
<th>Std. dev. (%)</th>
<th>Sharpe ratio</th>
<th>SMB</th>
<th>HML</th>
<th>MOM</th>
<th>PL</th>
<th>PC</th>
</tr>
</thead>
<tbody>
<tr>
<td>RM</td>
<td>6.84</td>
<td>17.82</td>
<td>0.38</td>
<td>0.28</td>
<td>−0.01</td>
<td>−0.12</td>
<td>0.05</td>
<td>−0.05</td>
</tr>
<tr>
<td>SMB</td>
<td>3.44</td>
<td>13.89</td>
<td>0.50</td>
<td>0.28</td>
<td>−0.12</td>
<td>0.05</td>
<td>−0.05</td>
<td>−0.05</td>
</tr>
<tr>
<td>HML</td>
<td>5.07</td>
<td>13.59</td>
<td>0.37</td>
<td>0.28</td>
<td>−0.01</td>
<td>0.05</td>
<td>−0.05</td>
<td>−0.05</td>
</tr>
<tr>
<td>MOM</td>
<td>8.57</td>
<td>18.28</td>
<td>0.47</td>
<td>0.28</td>
<td>−0.12</td>
<td>0.05</td>
<td>−0.05</td>
<td>−0.05</td>
</tr>
<tr>
<td>PL</td>
<td>−9.81</td>
<td>11.88</td>
<td>−0.83</td>
<td>0.28</td>
<td>−0.12</td>
<td>0.05</td>
<td>−0.05</td>
<td>−0.05</td>
</tr>
<tr>
<td>PC</td>
<td>13.15</td>
<td>14.22</td>
<td>0.92</td>
<td>0.28</td>
<td>−0.12</td>
<td>0.05</td>
<td>−0.05</td>
<td>−0.05</td>
</tr>
</tbody>
</table>

Panel C: Time-series regressions

\[ P_L = \alpha + \beta_M R_M + \beta_{SMB} S_{MB} + \beta_{HML} H_{ML} + \beta_{MOM} M_{OM} + \varepsilon_t \]

Model (1) \((\alpha)\) \((-7.46)\) \((-3.32)\) \((1.34)\) \((1.26)\) \(R^2\)

Model (2) \((\alpha)\) \((-3.64)\) \((-17.94)\) \((3.31)\) \((-142)\) \((-30.96)\) \(R^2\)

Panel D: Time-series regressions

\[ P_C = \alpha + \beta_M R_M + \beta_{SMB} S_{MB} + \beta_{HML} H_{ML} + \beta_{MOM} M_{OM} + \varepsilon_t \]

Model (1) \((\alpha)\) \((0.45)\) \((0.60)\) \((-0.25)\) \((0.79)\) \(R^2\)

Model (2) \((\alpha)\) \((0.20)\) \((0.65)\) \((-0.25)\) \((0.88)\) \((0.29)\) \(R^2\)

Intertemporal substitution of 0.05. These estimates are close to those in Kroencke (2017).\(^{17}\) Examining the goodness of fit in Panel D, we see that the main model has an \(R^2\) of 0.91 and the RMSE is 1.06% per year only. The J-test cannot reject the model at the 1% significance level, which suggests that the model prices the 24 test assets.

Turning to the power utility case in Column (1), we see that it shows slightly larger RMSE than the main model, but it works fairly well. Column (1) reports that the longevity factor is significantly priced at −4.47% and the estimated relative risk aversion is 23.91. Both are close to the estimates in Column (2). In fact, the success of a power utility can be deduced from the recursive preferences in Column (2). Since the estimated related risk aversion (\(\gamma\)) is very close to the reciprocal of the estimated elasticity of intertemporal substitution (1/\(\psi\)), the recursive preferences are very close to those of a power utility. This explains why the results in Columns (1) and (2) are very close.

The consumption-based models perform well in the cross-section of 24 test assets in Columns (1) and (2). In contrast, CAPM, in Column (3), and the Fama–French three-factor model, in Column (4), generate large pricing errors. Although the market factor is significantly priced, CAPM has an RMSE of 3.22%, together with an \(R^2\) as low as −0.01 in Column (3). The Fama–French three-factor model also shows a large RMSE of 2.34% and a moderate \(R^2\) of 0.48 in Column (4). Overall, the consumption-based models perform better than CAPM and the Fama–French three-factor model.

4. Cross-section of portfolios sorted by size and momentum

In the previous sections, we demonstrate the pricing power of longevity factor, especially for the momentum portfolios. But we still need to understand why longevity captures the cross-sectional return variations. To this end, in this section, we further examine the ability of longevity risk to explain the time-series and cross-sectional variation in returns on momentum portfolios, using the standard Fama–MacBeth regressions. The main test assets are 25 size and momentum sorted portfolios over July 1963 to December 2014, which are obtained from Kenneth French’s

\(^{17}\) The estimate of relative risk aversion appears to be smaller than the typical estimate in the literature. This is largely due to the fact that the unfiltered annual consumption data are very volatile. The standard deviation of unfiltered consumption is 2.62%, while it is only 1.34% in the original NIPA data. If we use the original NIPA consumption data, the main model implies a relative risk aversion of 38.93.
Table 2
This table presents the two-step GMM estimation with one-lag Newey–West adjustment of consumption-based CAPM (see, e.g., Eq. (28)). Factors include shocks to the time preferences, consumption growth rate, and the market portfolio. The unfiltered consumption data on nondurable goods and services are from Kroencke (2017). The shocks to time preferences are measured as the first-order difference of the weighted average period life expectancy. Test assets include six size and book-to-market sorted portfolios, six size and investment sorted portfolios, six size and operating profitability sorted portfolios, and six size and momentum sorted portfolios. Stock returns are adjusted by the consumption price index to convert into real returns when necessary. Column (1) presents estimates from a power utility, while Column (2) presents results from the Epstein–Zin recursive preferences. For comparison, Columns (3) and (4) present GMM estimates from CAPM and the Fama–French three-factor model. Panel A shows the coefficients ($b$) from the GMM estimation, and their t-statistics are in parentheses. * indicates that the coefficient is restricted by the model, not by the estimation. Panel B reports the implied price of risk ($\lambda$) for each factor, based on estimates in Panel A. Panel C presents the implied parameters, i.e., relative risk aversion ($\gamma$) and the elasticity of intertemporal substitution ($\psi$). Panel D provides statistics of goodness of fit, including $R^2$, root-mean-square errors (RMSE), and Hansen’s J-test of overidentification. $R^2$ is defined as one minus the ratio of the cross-sectional variance of the pricing errors to the cross-sectional variance of realized average portfolio returns, following Campbell and Vuolteenaho (2004). The annual data from 1964 to 2014 are used.

<table>
<thead>
<tr>
<th>Consumption CAPM</th>
<th>CAPM</th>
<th>Fama–French model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>Recursive preferences</td>
</tr>
<tr>
<td><strong>Panel A: Coefficients</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longevity ($b_1$)</td>
<td>-1</td>
<td>-1.30</td>
</tr>
<tr>
<td>Consumption ($b_2$)</td>
<td>23.91</td>
<td>25.42</td>
</tr>
<tr>
<td>Market ($b_{mb}$)</td>
<td>-0.30</td>
<td>2.62</td>
</tr>
<tr>
<td>SMB ($b_{smb}$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HML ($b_{hml}$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Panel B: Implied price of risk</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longevity ($\lambda_L$, %)</td>
<td>-4.47</td>
<td>-5.17</td>
</tr>
<tr>
<td>Consumption ($\lambda_C$, %)</td>
<td>1.72</td>
<td>1.82</td>
</tr>
<tr>
<td>Market ($\lambda_{mb}$, %)</td>
<td>2.61</td>
<td>8.74</td>
</tr>
<tr>
<td>SMB ($\lambda_{smb}$, %)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HML ($\lambda_{hml}$, %)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Panel C: Implied parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>23.91</td>
<td>25.13</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td><strong>Panel D: Goodness of fit</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.91</td>
<td>0.91</td>
</tr>
<tr>
<td>RMSE (%)</td>
<td>1.08</td>
<td>1.06</td>
</tr>
<tr>
<td>p-value ($f$)</td>
<td>0.46</td>
<td>0.41</td>
</tr>
</tbody>
</table>

website. These 25 size-momentum portfolios are sorted by the market capitalization and prior 11-month returns (skipping one month) at the end of the previous month.

4.1. Estimating mimicking portfolios of consumption and longevity risks

We first construct mimicking portfolios to track the consumption factor and longevity shocks. The mimicking portfolio approach is simple and easy to interpret, but one difficulty is that we have only annual data of consumption and longevity risks. Following Adrian et al. (2014), we first construct a mimicking portfolio of consumption (longevity) risk by projecting the consumption (longevity) risk onto a set of base asset returns at an annual frequency. Then we apply the same set of normalized weights to the base assets at a monthly frequency. This gives us the monthly mimicking portfolio returns.

Specifically, we first run the following regressions:

$$dC_t = \kappa_{0,C} + \kappa_{X,C}X_t + u_t,$$

$$dE_t = \kappa_{0,E} + \kappa_{X,E}X_t + z_t,$$

where $X_t$ represents the excess returns on base assets; $\kappa_{0,C}$, $\kappa_{X,C}$, $\kappa_{0,E}$, and $\kappa_{X,E}$ are coefficients. To avoid arbitrariness, we apply the same set of base assets to construct the mimicking portfolios for both consumption risk and longevity risk. We use the same set of base assets as in Adrian et al. (2014), i.e., the six Fama–French benchmark portfolios on size (small and big) and book-to-market (low, median, and high) in excess of the risk-free rate and the momentum factor. We choose these base assets to

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18 Ideally, we would use market-based measures to track the consumption factor and longevity risk. However, there is no financial assets directly linked to the consumption risk. Also, longevity products, such as longevity swaps, longevity bonds, or annuity products, started trading only recently and have a limited number of observations.

19 One concern is that those mimicking portfolios may be correlated, which leads to multicollinearity issues in regressions. We will address this issue later.
extract consumption and longevity information from asset returns as much as possible, especially given the high correlation between longevity risk and momentum presented in Fig. 1. However, one might be concerned that the mimicking longevity factor will artifactually do well for momentum portfolios. We address this concern in two ways. First, we test the pricing power of longevity risk with alternative assets in Section 5.2, in addition to the momentum portfolios. Second, as an alternative method, the GMM estimation of the consumption-based models in Section 3.3 avoids this issue.

Without loss of generality, we normalize the weights, $\kappa_x$, to sum to one for easy interpretation. The mimicking consumption portfolio return ($P_C$) and mimicking longevity portfolio return ($P_L$), are given by the fitted values, as follows:

$$P_C = \tilde{\kappa}_{x,C}X_t,$$

$$P_L = \tilde{\kappa}_{x,L}X_t,$$  \hspace{1cm} (31)

where $\tilde{\kappa}_x$ are normalized weights, e.g., $\tilde{\kappa}_{x,C} = \frac{\kappa_{x,C}}{\sum_{x,C}\kappa_{x,C}}$ and $\tilde{\kappa}_{x,L} = \frac{\kappa_{x,L}}{\sum_{x,L}\kappa_{x,L}}$. The annual data of base assets are obtained from Kenneth French’s website. The mimicking portfolio returns are estimated via ordinary least squares with a full sample or extending windows of data. For example, the normalized weights from a full-sample estimation over 1963–2014 are $\tilde{\kappa}_{x,C} = [-0.93, 0.83, 0.10, 0.24, 0.08, 0.41, 0.28]'$ and $\tilde{\kappa}_{x,L} = [0.53, -0.93, 0.25, -0.26, -0.37, 0.41, -0.61]'$. Overall, the mimicking portfolio tracks the longevity risk very well. We find that the correlation between the annual longevity risk and its mimicking portfolio returns is 0.31. We see that $P_L$ is negatively correlated with the momentum factor, while its correlation with the size and book-to-market factors is less clear. The correlation between the longevity risk and momentum profits is apparent in Fig. 1. Echoing this correlation, Fig. 2 plots the annual momentum factor (MOM) and returns on the mimicking portfolio of longevity risk ($P_L$). We see that the mimicking portfolio $P_L$ closely tracks the annual momentum profits (moving in opposite directions), including the huge momentum crash in 2009. Panels A and B of Table 1 present descriptive statistics of the portfolios mimicking consumption and longevity risks. The mimicking longevity factor ($P_L$) has an average return of $-9.81\%$ and a standard deviation of $11.88\%$ per year. This implies a Sharpe ratio of $-0.83$; the magnitude is much higher than the Sharpe ratios of the Fama–French three factors and the momentum factor. $P_L$ is highly correlated with the momentum factor with a correlation coefficient of $-0.82$, while its correlation with the Fama–French three factors is minor. The mimicking consumption factor ($P_C$) has an average return of $13.15\%$ and a standard deviation of $14.22\%$ per year. $P_C$ is highly correlated with the market factor, value factor ($HML$), and the longevity factor, but not the momentum factor.

Next, we apply the same set of normalized weights to the base assets at a higher frequency, i.e., monthly, to obtain the monthly mimicking portfolio returns. We will use the monthly mimicking factor returns in the cross-sectional asset pricing tests later. As preliminary tests, Panel C of Table 1 regresses the mimicking longevity factor against the benchmark Fama–French three factors or the Fama–French three factors plus the momentum factor. Model (1) shows that the Fama–French three factors explain little of the mimicking longevity risk. The $R^2$ is as low as $0.10$, and there is a large alpha of $-0.75\%$ per month. Adding the momentum factor to the Fama–French three-factor model, Model (2) shows that the momentum factor captures a significant part of longevity risk. Model (2) produces a much higher $R^2$ of $0.83$ and a smaller alpha of $-0.18$. In summary, Panel C of Table 1 suggests that longevity risk is unlikely to be captured by the Fama–French three factors and is highly correlated with the momentum factor. This gives us the first piece of evidence as to why longevity risk is able to explain the momentum profits.
Turning to the mimicking consumption factor, Panel D of Table 1 suggests that Fama–French three factors capture most of the consumption factor. For example, the $R^2$ in Model (1) is as high as 0.79. Adding the momentum factor in Model (2) only improves the $R^2$ slightly. This suggests that Fama–French three factors are reasonable proxies for the consumption factor.

### 4.2. Empirical models

Our first model is the consumption-based three-factor model in Eq. (25), as follows:

$$R_t = \alpha + \beta_M R_{Mt} + \beta_P PC_t + \beta_{PL} PL_t + \epsilon_R. \quad (33)$$

where $R_t$ is the excess return of test asset $i$ in month $t$; $R_{Mt}$ is the market factor in month $t$; $PC_t$ and $PL_t$ are the returns of the portfolios mimicking consumption risk and longevity risk in month $t$, respectively.

However, as noted in Table 1, the mimicking consumption portfolio ($PC$) is highly correlated with the market factor and the longevity factor. Such high correlations would generate multicollinearity concerns, if we include all three factors in the regressions. We address this issue via two alternative models. The GMM estimation results in the previous section show that the power utility specification works almost as well as the recursive preferences. This suggests that we can use the simple two-factor model in Eq. (26), which is based on the power utility, as follows:

$$R_t = \alpha + \beta_{PC} PC_t + \beta_{PL} PL_t + \epsilon_R. \quad (34)$$

This is our first alternative model. This two-factor model removes the market factor from the three-factor model and hence alleviates the multicollinearity problem.

Our second alternative model is the Fama–French three-factor model augmented with the longevity factor:

$$R_t = \alpha + \beta_M R_{Mt} + \beta_{SMB} SMB_t + \beta_{HML} HML_t + \beta_{PL} PL_t + \epsilon_R. \quad (35)$$

where $R_{Mt}$, $SMB_t$, and $HML_t$ are the Fama–French three-factor returns in month $t$. That is, we replace the market factor and the consumption factor in Model (33) with the Fama–French three factors. This can be viewed as a reduced form approximation of the consumption-based three-factor model. We propose to use the Fama–French three factors to proxy for the consumption risk instead of using the mimicking consumption portfolio for several reasons. First, this avoids any potential multicollinearity problem, because Panels B and C of Table 1 show that the mimicking longevity portfolio is not significantly correlated with the Fama–French three factors. Second, consumption data are notoriously noisy and hence less desirable in asset pricing tests, while beta pricing models often perform better. Empirically, as shown in Panels B and D of Table 1, Fama-French three factors seem to capture a large amount of consumption risk. Third, Fama–French factors are readily available at high frequencies, while consumption data are available at annual (quarterly) frequency only. We have to extrapolate the consumption factor to a higher frequency, and the performance deteriorates. Last, given the popularity of the Fama–French model, it would be interesting to see if longevity risk has any additional pricing power over the Fama-French factors.

Next, we estimate prices of risks using the Fama and MacBeth (1973) two-stage method over various models. In the first stage, we run the time-series regressions of Models (33)–(35) to estimate betas. Then we use betas as independent variables in the following cross-sectional regressions in each month $t$ to estimate the prices of risks in various models:

$$R_t = \gamma_0 + \gamma_M \hat{\beta}_{Mt} + \gamma_{PC} \hat{\beta}_{PCi} + \gamma_{PL} \hat{\beta}_{PLi} + \epsilon_t. \quad (36)$$

$$R_t = \gamma_0 + \gamma_{PC} \hat{\beta}_{PCi} + \gamma_{PL} \hat{\beta}_{PLi} + \epsilon_t. \quad (37)$$

$$R_t = \gamma_0 + \gamma_{SMB} \hat{\beta}_{SMBi} + \gamma_{HML} \hat{\beta}_{HMLi} + \gamma_{PL} \hat{\beta}_{PLi} + \epsilon_t. \quad (38)$$

We follow Shanken (1992) to adjust for the errors-in-variables problem when estimating the $\gamma$ terms (prices of risks). The adjusted cross-sectional $R^2$ is computed as in Jagannathan and Wang (1996). Lewellen et al. (2010) argue that it is easy to find a high cross-sectional $R^2$ when there is a strong factor structure. We follow Lewellen et al. (2010) to construct a sampling distribution of the adjusted $R^2$. Specifically, we bootstrap the time-series data of returns and factors by sampling with replacement. Then we estimate the adjusted $R^2$. We repeat these procedures 10,000 times and report the 5th and 95th percentiles of the sampling distribution. Following Lewellen et al. (2010), we add the traded portfolios as the test assets of the regressions to restrict the cross-sectional price of risk for a factor as the factor’s expected excess return.

### 4.3. Prices of risks

Table 3 presents the main results from the Fama-MacBeth regressions, using the standard Fama–French three-factor model and Models (33)–(35) with the monthly data. Panel A reports the price risks from the full-sample estimation. The mimicking portfolios and the betas are estimated from a full sample over July 1963 to December 2014. As expected, the Fama–French three factors perform poorly in explaining the 25 size-momentum portfolios. The intercept $\gamma_0$ in Column (1) is 0.74% per month, which is significant at the 1% level, together with a very low $R^2$ of 0.07. This implies that some of the 25 portfolios are mispriced by the Fama–French three-factor model. Examining the consumption-based three-factor model, we see Column (2) shows an adjusted $R^2$ of 0.80, which is in the right tail of its distribution. But even the 5th percentile of $R^2$ in Column (2) is as high as 0.63, which is higher than the 95th percentile of $R^2$ (0.34) generated by the Fama-French three-factor model in Column (1). This signals the good fit of this model. The longevity risk ($PL$) has a negative price of −0.83% per month, which is significant at the 1% level. The market factor and consumption factor are significantly priced, with a monthly price of 0.66% and 0.73%, respectively. But Column (2) shows

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20 Liew and Vassalou (2000) suggest that Fama–French three factors proxy for the macroeconomic variables.
Table 3
Cross-sectional regressions: main results.
This table presents Fama–MacBeth regressions using the excess returns of 25 portfolios sorted by size and momentum. Factors include the Fama–French three factors, the mimicking portfolio for consumption factor (PC), and the mimicking portfolio for longevity factor (PL). The factor betas, which are the independent variables in the regressions, are computed either over the full sample (full-sample betas in Panel A) or in extending windows (extending-window betas in Panel B). All coefficients are multiplied by 100. The t-statistics are in parentheses and adjusted for errors-in-variables, following Shanken (1992). The adjusted R² follows Jagannathan and Wang (1996). The 5th and 95th percentiles of the adjusted R² distribution from a bootstrap simulation of 10,000 times are reported in brackets. The sample period is from July 1963 to December 2014.

<table>
<thead>
<tr>
<th>Panel A: Full-sample betas</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>γ₀</td>
<td>0.74</td>
<td>0.09</td>
<td>0.18</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(5.85)</td>
<td>(1.92)</td>
<td>(1.44)</td>
<td>(0.49)</td>
</tr>
<tr>
<td>γ₉₅</td>
<td>−0.09</td>
<td>0.66</td>
<td>0.56</td>
<td>(−0.42)</td>
</tr>
<tr>
<td></td>
<td>(3.50)</td>
<td>(3.04)</td>
<td>(3.04)</td>
<td></td>
</tr>
<tr>
<td>γ₉₅M</td>
<td>0.24</td>
<td>0.24</td>
<td>(−1.80)</td>
<td>(1.80)</td>
</tr>
<tr>
<td>γ₉₅M</td>
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<td>0.36</td>
<td>(−2.39)</td>
<td>(2.61)</td>
</tr>
<tr>
<td>γ₉₅PC</td>
<td>0.73</td>
<td>0.80</td>
<td>(−4.49)</td>
<td>(4.55)</td>
</tr>
<tr>
<td>γ₉₅PL</td>
<td>−0.83</td>
<td>−0.84</td>
<td>−0.84</td>
<td>−0.80</td>
</tr>
<tr>
<td></td>
<td>(−6.50)</td>
<td>(−6.54)</td>
<td>(−6.17)</td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.07</td>
<td>0.80</td>
<td>0.81</td>
<td>0.91</td>
</tr>
<tr>
<td></td>
<td>[−0.07, 0.34]</td>
<td>[0.63, 0.88]</td>
<td>[0.55, 0.88]</td>
<td>[0.81, 0.94]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Extending-window betas</th>
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<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>γ₀</td>
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<td>0.14</td>
<td>0.40</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>(2.31)</td>
<td>(1.24)</td>
<td>(1.53)</td>
<td>(1.26)</td>
</tr>
<tr>
<td>γ₉₅</td>
<td>0.40</td>
<td>0.79</td>
<td>0.66</td>
<td>(1.43)</td>
</tr>
<tr>
<td></td>
<td>(2.96)</td>
<td>(2.70)</td>
<td>(2.70)</td>
<td></td>
</tr>
<tr>
<td>γ₉₅M</td>
<td>0.02</td>
<td>0.09</td>
<td>(0.11)</td>
<td>(0.56)</td>
</tr>
<tr>
<td>γ₉₅M</td>
<td>−0.18</td>
<td>0.20</td>
<td>(−0.75)</td>
<td>(0.98)</td>
</tr>
<tr>
<td>γ₉₅PC</td>
<td>−4.01</td>
<td>−4.47</td>
<td>(−1.04)</td>
<td>(−1.14)</td>
</tr>
<tr>
<td>γ₉₅PL</td>
<td>−0.85</td>
<td>−0.86</td>
<td>−1.06</td>
<td>(−3.41)</td>
</tr>
<tr>
<td></td>
<td>(−3.78)</td>
<td>(−4.55)</td>
<td>(−4.55)</td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>−0.00</td>
<td>0.94</td>
<td>0.94</td>
<td>0.86</td>
</tr>
<tr>
<td></td>
<td>[−0.12, 0.48]</td>
<td>[0.69, 0.99]</td>
<td>[0.46, 0.99]</td>
<td>[0.72, 0.92]</td>
</tr>
</tbody>
</table>

a marginally significant intercept of 0.09% per month (t-statistic=1.92), which might be due to the multicollinearity, given the high correlation between the mimicking consumption portfolio and the market factor, the mimicking longevity factor. We further tackle this multicollinearity issue in two ways (see Internet Appendix C for more details). First, following Menkhoff et al. (2012), we project the mimicking consumption portfolio on the market factor and the mimicking longevity portfolio and then use the orthogonalized component of consumption factor in the regressions. We find that the pricing error is insignificant. This signals the distortions due to multicollinearity. Second, we perform the partial least square (PLS) regressions. PLS regressions consider the correlation with dependent variables when extracting the key components from predictors (Kelly and Pruitt, 2013, 2015). We extract two predictor scores from the three factors, e.g., the market factor, the mimicking consumption portfolio, and the mimicking longevity portfolio. Then we use these two predictor scores in the Fama–MacBeth regressions. Again, we find the pricing error becomes insignificant.

The two-factor model in Column (3) produces results similar to those in Column (2), but it has an insignificant intercept of 0.18% per month (t-statistic=1.44). Finally, we see the Fama–French model augmented with the longevity factor in Column (4) provides the best fit, with an R² of 0.91 and a tiny intercept of 0.02% per month (t-statistic=0.49). Column (4) also shows reasonable estimates of the Fama–French three factors. Examining across Columns (2)–(4), we see all models perform well and produce very close estimates of longevity factor. Overall, the two-factor model, Column (3), performs comparably to the three-factor model, Column (2). Practically, the Fama–French model augmented with the longevity factor, Column (4), performs best. This is probably due to the fact that the Fama–French model augmented with the longevity factor avoids the multicollinearity problem.

Fig. 3 visualizes the performance of the Fama–French model and the Fama–French model augmented with longevity factor. We plot the fitted average return of each portfolio against its realized average return. The fitted return is computed using the beta estimates from a given model specification. The realized average return is the time-series average of the portfolio return over July 1963 to December 2014. If the model is correctly specified, then the fitted and realized average returns should be the same and lie on the 45-degree line through the origin. Panel (a) shows that the Fama–French model exhibits significant pricing errors. The fitted and realized average returns lie on a flat line, which implies that the Fama–French model overestimates (underestimates) the returns of the losers (winners) portfolio. In contrast, Panel (b) shows that the Fama–French model augmented with longevity factor successfully captures the momentum portfolios. The fitted and realized average returns closely lie on the 45-degree line in Panel (b).

To avoid the look-ahead bias, we also follow Ferson and Harvey (1999) to estimate the risk prices with extending windows in Panel B of Table 3. We require at least 20 annual observations to estimate the mimicking portfolios, so the cross-sectional regressions start in 1983. Panel B shows similar results as Panel A, except that the consumption factor becomes insignificant. Again, we see that longevity risk is negatively priced and captures the momentum profits very well (all intercepts in Columns (2)–(4) are insignificant).

4.4. Factor loadings of momentum portfolios on longevity risk

A negative price of longevity risk means that stocks that coarray positively with longevity risk (dEₜ) should have lower expected returns because these stocks have higher payoffs when dEₜ is high, i.e., when longevity increases. That is, these stocks provide hedging against increases in

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21 This is due to the multicollinearity problem.
Fig. 3. Fitted returns versus average realized returns. This figure shows realized average returns on the horizontal axis and fitted returns on the vertical axis for 25 portfolios sorted by size and momentum. For each portfolio, the realized average return is the time-series average of the portfolio returns, and the fitted return is from the corresponding model. The straight line is the 45-degree line from the origin. The sample period is from July 1963 to December 2014.
longevity. Similarly, stocks that covary negatively with $dE_i$ would have higher expected returns. Therefore, if longevity risk explains momentum profits, we should observe that past winners (losers) have negative (positive) betas on the mimicking longevity portfolio $PL$. This implies that losers provide hedging against longevity risk, while winners hedge against mortality risk. In this section, we examine the factor loadings of 25 size-momentum sorted portfolios on the longevity factor. We report the full-sample time-series regression results from the Fama–French model augmented with the longevity factor in Table 4. We choose to report the results from Model (35) instead of Models (33) and (34), because Model (35) avoids the multi-collinearity problem and provides more reliable estimates of factor loadings.

Inspecting the betas on $PL$ gives rise to several observations. First, within each size quintile, $\beta_{PL}$ monotonically decreases from the losers to the winners portfolio. This alleviates the concern about whether longevity risk is a characteristic (Daniel and Titman, 1997). Notably, the loadings on $PL$ are significantly different between the past winners and losers portfolios in all size quintiles. In fact, the difference in $\beta_{PL}$ between winners and losers is slightly higher in the two largest size quintiles. Second, the losers always have positive betas on $PL$, while the winners always have negative ones, which is consistent with the prediction above. Third, if we move from winners to losers quintiles, the magnitude of the betas of losers is much larger than that of winners. This is consistent with the empirical finding that the losers portfolio contributes more to the momentum profits than the winners portfolio (see, e.g. Jegadeesh and Titman, 2001). Fourth, overall, we see that 23 out of 25 portfolios have significant $\beta_{PL}$ at the 1% level.

Turning to other estimates in Table 4, we see that most portfolios have very small alphas. Winners have lower loadings on the market factor and the value factor, which suggests that these two factors are unlikely to explain the momentum profits. Winners do have slightly higher loadings on the size factor, while smaller stocks have higher exposure to the size factor. Overall, consistent with the literature, there is little evidence that the Fama–French three factors help to explain the momentum profits.

### Table 4

Time-series regressions of the Fama–French model augmented with the longevity risk factor.

<table>
<thead>
<tr>
<th></th>
<th>Losers 2</th>
<th>3</th>
<th>4</th>
<th>Winners</th>
<th>Losers 2</th>
<th>3</th>
<th>4</th>
<th>Winners</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha$</td>
<td>diff</td>
<td>t-statistic</td>
<td>diff</td>
<td>$\beta_{PL}$</td>
<td>diff</td>
<td>t-statistic</td>
<td>diff</td>
</tr>
<tr>
<td>Small</td>
<td>$-0.26$</td>
<td>0.03</td>
<td>0.12</td>
<td>0.17</td>
<td>0.33</td>
<td>0.59</td>
<td>$-1.85$</td>
<td>0.34</td>
</tr>
<tr>
<td>2</td>
<td>$-0.13$</td>
<td>0.07</td>
<td>0.03</td>
<td>0.11</td>
<td>0.15</td>
<td>0.28</td>
<td>$-1.01$</td>
<td>0.91</td>
</tr>
<tr>
<td>3</td>
<td>0.13</td>
<td>0.09</td>
<td>0.04</td>
<td>$-0.09$</td>
<td>0.14</td>
<td>0.01</td>
<td>1.09</td>
<td>1.00</td>
</tr>
<tr>
<td>4</td>
<td>0.14</td>
<td>0.15</td>
<td>0.05</td>
<td>0.02</td>
<td>0.02</td>
<td>$-0.12$</td>
<td>1.10</td>
<td>1.81</td>
</tr>
<tr>
<td>Large</td>
<td>0.20</td>
<td>0.30</td>
<td>$-0.03$</td>
<td>$-0.09$</td>
<td>$-0.08$</td>
<td>$-0.28$</td>
<td>1.37</td>
<td>3.70</td>
</tr>
</tbody>
</table>

$\beta_{PL}$ is the longevity risk factor, and $t$-statistic is the $t$-statistic of the estimated factor loadings.
Internet Appendix D presents the results from Models (33) and (34), which are qualitatively similar to those in Table 4. For example, losers (winners) have positive (negative) loadings on the longevity factor, and \( \beta_{PL} \) monotonically decreases from the losers to the winners portfolio. Meanwhile, we find that winners have lower loadings on the market factor and consumption factor than losers, which means that these two factors cannot explain the momentum returns.

5. Robustness checks

5.1. Subperiod analysis

To further examine the persistence of the pricing power of the longevity factor, we split the whole sample into two subperiods, i.e., 1963–1986 and 1987–2014, and run the full-sample Fama–MacBeth regressions for 25 portfolios sorted by size and momentum. In Table 5, Columns (1)–(4) show the results from 1963–1986, while Columns (5)–(8) show the results from the second subperiod, 1987–2014.

Examining results from Columns (1) and (5), we see that the benchmark Fama–French model has difficulties in explaining the momentum in both periods. The pricing error is 0.73% per month (t-statistic=5.84) in the first period and 0.69% per month (t-statistic=3.45) in the second period. Investigating Columns (2)–(4) and (6)–(8), we see that longevity factor captures the momentum profits in both periods, with a negative price of risk ranging from −0.89% to −0.64% per month. The consumption-based three-factor model and its two variants perform well.

5.2. Alternative test assets

Lewellen et al. (2010) suggest expanding the set of test assets when some factors seem to explain nearly all of the variation in returns. If longevity risk is an important state variable, it should be priced in other assets as well. Thus we use other portfolios to check the robustness of the models in this section.

Besides 25 size and momentum sorted portfolios, we use 25 portfolios sorted by size and book-to-market (BM), 25 portfolios sorted by size and investment, and 25 portfolios sorted by size and operating profitability. The data of these 100 portfolios come from Kenneth French’s website. The sample period is from July 1963 to December 2014. Novy-Marx (2015) suggests that earnings momentum can subsume the return momentum. To test whether longevity risk is able to price the earnings momentum, we include portfolios formed by size and earnings surprise as additional test assets. Specifically, we add 25 portfolios formed by size and earnings surprise, where earnings surprise is measured as the standardized unexpected earnings (SUE), following Novy-Marx and Velikov (2016). The sample period for 25 size and earnings surprise sorted portfolios is from January 1974 to December 2014.

Table 6 reports the full-sample Fama–MacBeth regression results. For brevity, we report the results of testing these portfolios jointly. Given data availability, we first consider 100 portfolios (denoted as “100 portfolios”) over July 1963 to December 2014 in Panel A and then all 125 portfolios (denoted as “125 portfolios”) over January 1974 to December 2014 in Panel B. Table 6 shows results similar to those in Table 3. The Fama–French model demonstrates large pricing errors in Columns (1) and (5), while all other models perform well (with insignificant pricing errors) in Columns (2)–(4) and (6)–(8). Overall, we see that longevity risk is negatively priced in the cross-section of various test assets, while consumption risk is positively priced.

5.3. Annual estimation

Since consumption and longevity data are available only annually, in Section 4.1, we apply the weights of mimicking portfolios obtained from annual regressions to the monthly
Table 6
Robustness checks: alternative test assets.
This table presents Fama–MacBeth regressions using the excess returns of various test assets with full-sample regressions. Panel A tests 100 portfolios jointly, which include 25 size and book-to-market sorted portfolios, 25 size and investment sorted portfolios, 25 size and operating profitability sorted portfolios, and 25 size and momentum sorted portfolios. The data of test assets are obtained from Kenneth French’s website. Panel B further adds a set of 25 portfolios formed by size and earnings surprise, where earnings surprise is measured as the standardized unexpected earnings (SUE). SUE follows Nonsy-Mars and Velikov (2016). Factors include the Fama–French three factors, the mimicking portfolio for consumption factor (PC), and the mimicking portfolio for longevity factor (PL). All coefficients are multiplied by 100. The t-statistics are in parentheses and adjusted for errors in variables, following Shanken (1992). The adjusted $R^2$ follows Jagannathan and Wang (1996). The 5th and 95th percentiles of the adjusted $R^2$ distribution from a bootstrap simulation of 10,000 times are reported in brackets. Due to data availability, the sample period for Panel A is from July 1963 to December 2014; the sample period for Panel B is from January 1974 to December 2014.

<table>
<thead>
<tr>
<th>Panel A: 100 Portfolios</th>
<th>Panel B: 125 Portfolios</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_0$</td>
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</tr>
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<td>(5.31)</td>
</tr>
<tr>
<td>$\gamma_1$</td>
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<tr>
<td>(-1.19)</td>
<td>(-0.80)</td>
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<tr>
<td>$\gamma_2$</td>
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<tr>
<td>(0.01)</td>
<td>(0.12)</td>
</tr>
<tr>
<td>$\gamma_3$</td>
<td>(2.75)</td>
</tr>
<tr>
<td>(3.75)</td>
<td>(3.67)</td>
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<tr>
<td>$\gamma_{PL}$</td>
<td></td>
</tr>
<tr>
<td>(-0.82)</td>
<td>(-0.85)</td>
</tr>
<tr>
<td>(-0.81)</td>
<td>(-0.82)</td>
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<tr>
<td>(-0.79)</td>
<td>(-0.86)</td>
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<tr>
<td>$\gamma_{PC}$</td>
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<tr>
<td>(4.92)</td>
<td>(3.97)</td>
</tr>
<tr>
<td>$\gamma_{PL}$</td>
<td></td>
</tr>
<tr>
<td>(-6.38)</td>
<td>(-5.66)</td>
</tr>
<tr>
<td>(-6.23)</td>
<td>(-5.30)</td>
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<td>(-6.19)</td>
<td>(-5.72)</td>
</tr>
<tr>
<td>$R^2$</td>
<td></td>
</tr>
<tr>
<td>0.19</td>
<td>0.17</td>
</tr>
<tr>
<td>0.57</td>
<td>0.41</td>
</tr>
<tr>
<td>0.57</td>
<td>0.41</td>
</tr>
<tr>
<td>0.85</td>
<td>0.70</td>
</tr>
<tr>
<td>[0.04, 0.46]</td>
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<tr>
<td>[0.34, 0.75]</td>
<td>[0.21, 0.62]</td>
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<td>[0.26, 0.72]</td>
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</tr>
<tr>
<td>[0.72, 0.88]</td>
<td>[0.54, 0.77]</td>
</tr>
</tbody>
</table>

In $m_{x,t} = a_k + b_kK_t$, where $D_{x,t}$ is the number of deaths for age $x$ in year $t$, $E_{x,t}$ is the corresponding exposure, $m_{x,t}$ is the central death rate, $a_k$ and $b_k$ are two age-specific parameters, and $K_t$ is the mortality index. The model estimates this underlying mortality index, using a panel of mortality data across selected age groups. The parameters $a_k$, $b_k$, and $K_t$ are estimated iteratively using the maximum likelihood method (see Internet Appendix F for more details). Then our alternative measure of longevity risk is the innovations in the mortality index, i.e., the mortality risk:

$$dK_t = K_t - K_{t-1}. \quad (41)$$

A negative shock to the mortality index implies a positive shock to longevity, so these two longevity measures have opposite signs of the price of risk.

The mortality index is estimated via a full sample of data, over 1963–2014. It decreases over time, reflecting the general trend of rising life expectancy in this period. Empirically, we find that mortality risk ($dK_t$) has a mean of –1.30 (corresponding to an increase of 0.14 years in life expectancy), a standard deviation of 1.31, and a negative first-order autocorrelation of –0.13. ADF unit root tests show that mortality shocks ($dK_t$) are stationary. We find that mortality risk is highly correlated with the longevity risk (a correlation of –0.99). Next, we construct a mimicking mortality portfolio, similar to the mimicking longevity portfolio in Eqs. (30) and (32). The mimicking mortality portfolio has an annual return of 9.29% with a standard deviation of 12.28%. This mimicking mortality portfolio is highly correlated with the momentum factor (a correlation of 0.87). Internet Appendix G shows more details about the mortality risk and its mimicking portfolio returns.
We first replace the mimicking longevity portfolio with the mimicking mortality portfolio and perform the Fama–MacBeth two-stage regressions in Table 8. Panel A tests 25 size and momentum sorted portfolios over July 1963 to December 2014; Panel B tests 100 portfolios over July 1963 to December 2014; Panel C tests 125 portfolios over January 1974 to December 2014. Table 8 presents results similar to those in Tables 3 and 6. For example, we see the mortality factor is significantly priced in all cases, with a positive price ranging from 0.7% to 0.87% per month. In fact, even the magnitude of the price of mortality risk is very close to the price of longevity shown in Tables 3 and 6. Consumption risk is also positively priced in all cases. Overall, we see the consumption-based three-factor model, two-factor model, and the Fama–French model augmented with the mortality factor have insignificant pricing errors for those test portfolios. The Fama–French model augmented with the mortality factor performs best.

5.5. International evidence: UK markets

Momentum is observed in international markets and is particularly strong in the UK markets (Rouwenhorst, 1998; Griffin et al., 2003; Chui et al., 2010; Fama and French, 2012; Asness et al., 2013; Bali et al., 2013). In this section, we perform a similar analysis for the UK markets as further robustness checks. We follow Gregory et al. (2013) and use their datasets on UK stock markets, which include 6 size and book-to-market sorted portfolios, the Fama–French three factors, the momentum factor, and 20 size and momentum sorted portfolios. To minimize the impact of illiquidity and transaction costs associated with tiny stocks, we focus on the largest four size groups of those datasets, which cover Financial Times Stock Exchange 350 stocks only. The annual data of exposure and mortality rates are obtained from the Human Mortality Database. As in the previous analyses, we measure longevity risk as innovations of the weighted average period life expectancy or innovations of the mortality index. Then we construct the mimicking portfolios for these two measures. Due to data availability, the sample period is from January 1981 to December 2013. The annual consumption (including nondurable goods and services consumption) and population (aged 16 and over) data are obtained from the Office for National Statistics. Due to the structural change of consumption data in 1985 and data timing conventions, we use the consumption growth data over 1986–2014 to match with the returns of base assets over 1985–2013 when constructing the mimicking portfolio in Eq. (29), but we apply the same set of weights to compute the mimicking consumption portfolio returns over 1981–2013 in Eq. (31).

Table 9 reports the Fama–MacBeth two-stage regressions. Column (1) shows that the standard Fama–French three-factor model generates a large pricing error of 0.90% per month (t-statistic=2.04), together with a low $R^2$ of
Table 8

Robustness checks: alternative longevity measure.

This table presents full-sample Fama–MacBeth regressions of various test portfolios, using an alternative measure of longevity factor. We measure longevity risk as the innovations in the mortality index (Δk), i.e., the mortality risk. The mortality index is computed as in Brouhns et al. (2002). Factors include the Fama–French three factors, the mimicking portfolio for consumption factor (PC), and the mimicking portfolio for mortality factor (PM). Panel A tests 25 size and momentum sorted portfolios. Panel B tests 100 portfolios jointly, which include 25 size and book-to-market sorted portfolios, 25 size and investment sorted portfolios, 25 size and operating profitability sorted portfolios, and 25 size and momentum sorted portfolios. The data of test assets are obtained from Kenneth French’s website. Panel C further adds a set of 25 portfolios formed by size and earnings surprise, where earnings surprise is measured as the standardized unexpected earnings (SUE). SUE follows Novy-Mark and Veldkamp (2016). All coefficients are multiplied by 100. The t-statistics are in parentheses and adjusted for errors in variables, following Shanken (1992). The adjusted R2 follows Jagannathan and Wang (1996). The 5th and 95th percentiles of the adjusted R2 distribution from a bootstrap simulation of 10,000 times are reported in brackets. Due to data availability, the sample period for Panels A and B is from July 1963 to December 2014; the sample period for Panel C is from January 1994 to December 2014.

| Panel A: 25 Portfolios | | Panel B: 100 Portfolios | | Panel C: 125 Portfolios |
|------------------------|----------------|------------------------|------------------------|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| γ₀ | 0.74 | -0.03 | 0.28 | -0.02 | 0.55 | -0.25 | 0.17 | 0.02 | 0.61 | -0.19 | 0.31 | -0.02 |
| | (5.85) | (-0.35) | (1.01) | (-0.66) | (6.17) | (-1.66) | (0.47) | (0.62) | (5.31) | (-1.12) | (0.76) | (-0.33) |
| γₘ | -0.09 | 0.78 | 0.59 | 0.00 | 0.06 | 0.51 | 0.17 | 0.22 | 0.03 | 1.01 | 0.63 |
| | (-0.42) | (3.75) | (3.20) | (0.01) | (3.67) | (2.76) | (0.18) | (0.20) | (0.12) | (3.48) | (2.92) |
| γₓₐₓ | 0.24 | 0.23 | 1.77 | 1.44 | 1.56 | 0.20 | 0.42 | 1.54 | 0.22 | 0.25 | (1.75) |
| | (1.80) | | (2.90) | | | | | | | | |
| γₓₚₚ | -0.46 | 0.40 | 0.26 | 0.42 | 0.26 | 0.45 | 0.45 | 0.45 | 0.45 | |
| | (-2.39) | | (2.90) | | | | | | | |
| γₓₓ | 0.85 | 0.70 | 0.81 | 0.42 | 0.87 | 0.70 | 0.77 | 0.87 | 0.67 | 0.84 | |
| | (4.47) | (3.00) | | (5.94) | | | | | | | |
| γₓₙ | 0.83 | 0.72 | 0.79 | 0.81 | 0.87 | 0.70 | 0.77 | 0.87 | 0.67 | 0.84 | |
| | (5.83) | (4.20) | | (5.94) | | | | | | | |
| R² | 0.07 | 0.64 | 0.64 | 0.85 | 0.19 | 0.39 | 0.37 | 0.78 | 0.17 | 0.25 | 0.23 | 0.61 |
| | [-0.07, 0.34] | [0.35, 0.82] | [0.24, 0.81] | [0.70, 0.91] | [0.04, 0.46] | [0.13, 0.67] | [0.08, 0.64] | [0.62, 0.84] | [0.03, 0.43] | [0.06, 0.53] | [0.03, 0.51] | [0.44, 0.72] |

6.1. Equity durations

Previously, we empirically establish that losers and winners have different exposures to longevity risk. That is, losers (winners) provide hedging against longevity (mortality) risks, which drives the pricing power of longevity risk (mortality) risks. We further explore the economic mechanism behind this result.

When we add the longevity risk factor to the Fama–French model, Column (4) shows improved results. For example, we see an insignificant impact of 0.05 percent on equity returns. Adding the longevity factor also captures the large market premiums in the UK markets.
Clearly, losers plot in flow ration the match microstructure 1972 internet the book growth 2013. The Robustness Table This factors include the Fama–French three factors and the mimicking portfolio for longevity risk (PL), which is measured as innovations in the weighted average period life expectancy (denoted as dE) in Columns (2)-(4) or innovations in mortality index (denoted as dK) in Columns (5)-(7). Both measures are estimated over the full sample. The factor betas, which are the independent variables in the regressions, are computed over the full sample. All coefficients are multiplied by 100. The t-statistics are in parentheses and adjusted for errors in variables, following Shanken (1992). The adjusted R² follows Agannathan and Wang (1996). The 5th and 95th percentiles of the adjusted R² distribution from a bootstrap simulation of 10,000 times are reported in brackets. The sample period is from January 1981 to December 2013.

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<td>0.86</td>
<td>0.82</td>
<td>0.72</td>
<td>0.71</td>
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<td>[0.19, 0.81]</td>
<td>[0.18, 0.77]</td>
<td>[−0.08, 0.57]</td>
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book equity grows at the rate of sales growth (SGR). They further assume that SGR and ROE follow two separate first-order autoregressive processes. To avoid the seasonality issue, we estimate these two processes with the annual Compustat data and then convert the estimates into the quarterly frequency. Other data are from the quarterly Compustat. We project the cash flows for the next T=40 quarters and then compute durations from Eq. (42) (see Internet Appendix H for details). The sample period is from 1972 to 2014.

Next, we compute durations for 10 momentum portfolios. Similar to Jegadeesh and Titman (2001), at each month, we sort all common stocks from NYSE/Amex/Nasdaq into 10 portfolios, based on their cumulative returns over the previous 11 months. To avoid microstructure noise, we exclude penny stocks. Next, we match the duration estimates with the stock price data, assuming a three-month reporting lag. Then we compute the equally weighted duration for each portfolio. We examine the portfolio durations during and after portfolio formation.

Fig. 4 presents the durations of winners and losers during portfolio formation and the subsequent four quarters. We see that during the portfolio formation period, losers have a much longer duration (18.31 years) than winners (15.99 years). In other words, the losers portfolio has a duration of 2.33 years longer than the winners portfolio at the beginning of the holding period. This is due to the cash flow effect, because winners have higher dividend growth in the recent past and the near future than losers.24 Fig. 5 plots the quarterly dividend growth rates for winners and losers four quarters before and after portfolio formation. Clearly, winners experience much higher dividend growth one year before and after the portfolio formation. Also, Fig. 4 shows that the duration of losers (winners) decreases (increases) after the portfolio formation. The duration spread between losers and winners decreases to 0.76 years four quarters after portfolio formation. Fig. 5 also shows that winners and losers have about the same dividend growth four quarters after portfolio formation. This is consistent with the empirical finding that momentum profits exist mainly within one year after portfolio formation. Hence, Fig. 4 confirms our prediction that losers have longer equity durations and are preferred by investors who are facing a positive longevity shock. As stocks with longer durations have lower expected returns (see, e.g., Dechow et al., 2004; Lettau and Wachter, 2007; Da, 2009; Weber, 2018), losers underperform. That is, losers (winners) provide hedging against longevity (mortality) risks through the duration channel.

We see that prior losers have longer durations. However, this only implies that overall, losers have lower expected returns than winners. We need an additional mechanism to generate time-varying momentum profits, especially the large losses. This is provided by an important feature of longevity risk, its time-varying nature. As longevity risk varies over time, agents’ preferences for longer or shorter duration stocks change as well. This creates time-varying momentum profits. For example, momentum profits should be low when there is a spike in unexpected longevity. Let us examine the largest momentum crash, which is in 2009. The life expectancy is 80.88, 80.93, and 81.26 years in 2007, 2008, 2009, respectively. This pattern is also evident from the hospital admission rate of the whole population.25 Based on Hospital Statistics from the American Hospital Association (2016), the hospital admission rate is computed as total number of hospital admissions over the population.

24 Chen (2017) finds similar results for value and growth stocks, i.e., growth stocks have lower cash flow growth rates.

25 The hospital admission rate is computed as total number of hospital admissions over the population.
admission rate in 2007, 2008, and 2009 is 12.30%, 12.32%, and 12.20%, respectively. Note that 2008 is the financial crisis year, and life expectancy did not increase much (only 0.05 years, probably for economic reasons) in 2008 (the hospital admission rate increases slightly in 2008). But there is a big increase of life expectancy one year after the financial crisis (the hospital admission rate drops in 2009). It increases by 0.33 years in 2009, which maps very well the huge loss of momentum strategy in 2009. In fact, as shown in Fig. 1, longevity risk closely follows the three largest momentum crashes over the sample period, i.e., in 1975, 2003, and 2009.\footnote{There is a slight misalignment in 2003 due to the data timing conventions, e.g., longevity risk is computed at the mid-year while stock returns are computed over a year.} Longevity risk is unusually high in these three years, coinciding with the largest losses of momentum strategy.

6.2. Frequency domain analysis: short-run and long-run consumption risks

We first provide some statistical evidence from the frequency domain analysis.\footnote{Recently, Dew-Becker and Giglio (2016) applied frequency analysis to derive the frequency-specific risk prices for leading models.} Fig. 6 plots the spectral density of the longevity risk (dE), momentum factor, the Fama–French three factors, and the cross-spectrum of the longevity risk and the momentum factor. Panel (a) shows that the longevity risk has a very low frequency component with a frequency of 0.24 (a period of 26 years) and another business cycle frequency component with a frequency of 2.30–2.78 (a period of 2.26–2.74 years).\footnote{The low frequency component is likely to be driven by changes in the birth rate at the generational frequency, e.g., baby booms.} Panel (b) shows that the momentum factor also has a business cycle component with a frequency of 2.05–2.30 (a period of 2.74–3.06 years). Panels (a) and (b) suggest that the longevity risk and the momentum factor share a common business cycle component with a frequency of 2.30 (a period of 2.74 years), which is exactly shown in the cross-spectrum graph in Panel (c). Inspecting the Fama–French three factors, we see that the market factor has a business cycle component with a frequency of 1.81 (a period of 3.47 years); the size factor has a low frequency component with a frequency of 0.48 (a period of 13 years); the value factor has a business cycle component with a frequency of 1.33 (a period of 4.73 years). Frequency domain analysis provides us another way to understand the success of longevity risk in explaining the momentum profits: they have a common frequency component, which is at the business cycle frequency. This is consistent with the earlier findings that momentum appears to be procyclical (Chordia and Shivakumar, 2002; Cooper and Hameed, 2004). Frequency domain analysis also illustrates the inability of the Fama–French three factors to explain the momentum profits, as they do not have a common frequency component.

Frequency domain analysis shows that although longevity risk has a long-run component and a short-run component, only the short-run component is shared with the momentum factor. From the consumption risk perspective, this implies that only the short-run consumption risk matters for momentum factor. To further test this implication, we estimate the short-run and long-run consumption risks, following Bansal et al. (2016) and Li and Zhang (2017). Specifically, we regress consumption growth at year t+1 against the natural logarithm of aggregate dividend-price ratio and real risk-free rate at year t to compute the expected consumption growth rate. Next, we
fit an AR(1) process to the expected consumption growth. 

We then recover the short-run and long-run consumption risks. The annual data of aggregate dividend-price ratio and real risk-free rate are from Robert Shiller’s website. The unfiltered consumption data are from Kroencke (2017). The sample period is 1964–2014.

Fig. 7 plots the estimated short-run consumption risk and longevity risk. Clearly, longevity risk and the short-run consumption risk move in opposite directions. Indeed, we find that longevity risk is negatively associated with the short-run consumption risk, with a correlation coefficient of −0.27, while it is positively related to the long-run consumption risk, with a correlation coefficient of 0.11. Since both short-run and long-run risks are positively priced under the standard assumptions in Bansal and Yaron (2004), this further confirms the finding from frequency domain analysis that longevity risk contains short-run consumption risks, which explain the momentum factor, not the long-run component.

6.3. Decomposing longevity risk

We previously showed that longevity can affect time preferences directly via the time-preference discount rate channel or indirectly through the income inequality channel. One might wonder which channel is more important. To differentiate these two channels, we project longevity risk on income inequality risk to compute the component predicted by income inequality risk and the residuals. Then we construct the mimicking portfolio for the residuals as in Eqs. (30) and (32) (denoted as resid, which captures the direct channel). We compute the difference between the mimicking longevity portfolio and mimicking residual portfolio as the income inequality risk (denoted as II, the indirect channel). Last, we perform Fama–MacBeth regressions to test asset pricing power of these two components.

We use income data from the US Census Bureau (Table A-2: Selected measures of household income dispersion). We first compute the annual income growth rates for the 90th and 50th income percentiles and measure income growth dispersion as the difference between these two growth rates. Then, income inequality risk is defined as the first-order difference of income growth dispersion. We use full-sample estimation in Fama–MacBeth regressions. We use 125 test portfolios, including 25 size-momentum portfolios, 25 size-BM portfolios, 25 size-investment portfolios, 25 size-profitability portfolios, and 25 size-SUE portfolios. The sample period is 1974–2014.

Consistent with the literature, we find that longevity decreases with the income inequality, with a correlation of −0.18. Table 10 reports results from the Fama–MacBeth regressions, using the Fama–French three-factor

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Fig. 5. Dividend growth rates of winners and losers around portfolio formation. This figure plots quarterly dividend growth rates of winners and losers four quarters before and after portfolio formation. At each month, we sort all common stocks (excluding penny stocks) from NYSE/Amex/Nasdaq into 10 portfolios, based on their cumulative returns over the previous 11 months, skipping one month. Winners (losers) are stocks in the highest (lowest) decile. We compute quarterly dividend growth rate of each stock and then compute equally weighted dividend growth rate for each portfolio. The sample period is from 1972 to 2014.
Fig. 6. Spectral analysis. Fig. (a), (b), (d), (e), and (f) plot the spectral density of longevity risk ($dE$), momentum factor ($MOM$), market return ($MKT$), size factor ($SMB$), and value factor ($HML$), respectively, against frequency. Fig. (c) plots the cross-spectrum of longevity risk ($dE$) and momentum factor ($MOM$) against frequency.
model, the consumption-based three-factor model, the two-factor model, and the Fama–French model augmented with longevity factor. Columns (3), (5), and (7) shows that both income inequality component and the residual component are significantly priced with a negative price. Taking the consumption-based three-factor model as an example, income inequality component has a price of –0.38% (t-statistic=–3.57) per month, while the residual component has a price of –0.42% (t-statistic=–2.15) per month. The magnitudes of these two components are about the same in Columns (3) and (5), but the residual component seems to be much larger than the income inequality component in Column (7). In sum, both channels show a sizable price of risk.

6.4. Investigating the pricing kernel

Jegadeesh and Titman (1993) decompose the potential sources of momentum profits into three parts (see Eq. (3) on page 72): (1) the cross-sectional dispersion in unconditional expected returns, (2) the time-serial covariance of pricing factors, and (3) the average serial covariance of the idiosyncratic components of stock returns. The first two components imply risk-based explanations for momentum profits, while the last source implies market inefficiency. Jegadeesh and Titman (1993, 2002) show that the first component is not the reason for momentum profits, as stocks have similar unconditional returns. Jegadeesh and Titman (1993) further note that "the serial covariance
of 6-month returns of the equally weighted index is negative (−0.0028)" (see page 73), which implies that the second term cannot contribute to the momentum profits either. That is, momentum profits cannot be explained by the market factor. Their finding is not surprising, given the poor performance of CAPM.

Since we suggest that longevity risk contributes to most momentum profits, we need to show that the pricing factors in our model are indeed positively serially correlated. To test this hypothesis, we first aggregate the multiple pricing factors into one factor, i.e., the pricing kernel. Let \( R_{t+1} \) be the return of asset \( i \) at time \( t + 1 \), which satisfies a \( k \)-factor linear asset pricing model, as follows:

\[
E[R_{t+1}] = \alpha + \lambda' \Sigma_f^{-1} \text{Cov}(f_{t+1}, R_{t+1}) = \alpha + \phi' \text{Cov}(f_{t+1}, R_{t+1}),
\]

where \( f \) are pricing factors, \( \lambda \) are prices of factor risks, \( \Sigma_f \) is the covariance matrix of factors \( f \), and \( \phi = \Sigma_f^{-1} \lambda \). Then we can define the pricing kernel as

\[
= \frac{1}{\alpha} \left[ 1 - \phi'(f_{t+1} - \mu_f) \right].
\]

where \( \mu_f \) are unconditional means of the factors.

We use Eq. (44) to construct a pricing kernel from the three-factor model, two-factor model, and the Fama–French model augmented with the longevity factor. Since the momentum portfolios are constructed over previous 11-month returns, we use the annual factor data to construct the time series of the annual pricing kernel. We find that the pricing kernel constructed from the consumption-based three-factor model, two-factor model, and the Fama–French model augmented with the longevity factor is indeed positively serially correlated, with a correlation coefficient of 0.14, 0.15, and 0.11, respectively. For comparison, we also compute the annual pricing kernel implied by CAPM or the Fama–French three-factor model. We find that the pricing kernels from CAPM and the Fama–French three-factor model are negatively serially correlated, with a correlation coefficient of −0.04 and −0.04, respectively. This explains both the failures of CAPM and the Fama–French three-factor model and the success of longevity risk in capturing the momentum profits. Longevity risk does represent systematic risk underlying the momentum strategy.

7. Conclusion

Time-preference shocks affect agents' preferences for assets with different durations, which represent a systematic risk on agents' intertemporal consumption and investment choices. Unexpected shocks to life expectancy are important sources of time-preference shocks. For example, longevity can affect time preferences directly via the time-preference discount rate or indirectly through the income inequality channel. We model longevity risk as a stochastic time-preference shock process in the recursive preferences setting. We show that the consumption-based model implies a linear three-factor model, which includes the longevity risk (time-preference shocks), the consumption growth rate, and the market portfolio, where longevity is negatively priced. Empirically, we find that the consumption-based three-factor model and its two variants, i.e., the two-factor model and the Fama–French three-factor model augmented with the longevity risk, are able to explain the cross-sectional return variations generated by many well-known portfolios. Notably, we find that longevity risk closely tracks the momentum profits. We find that losers have lower dividend growth and hence longer durations than winners. Thus, the previous losers (winners) provide hedging against the longevity (mortality) risk and consequently have lower (higher) expected returns. In addition, agents' preferences for longer or shorter duration stocks change over time as longevity risk varies. We find that longevity risk explains most momentum profits, including the large momentum crashes observed in the data. We find that longevity risk shares a common business cycle component with the momentum factor. This short-run risk component explains the momentum factor. Our cross-sectional results highlight the importance of including time-preference shocks in asset pricing, as demonstrated in Albuquerque et al. (2016).

References


