A Theory of Endogenous Coalition Formation in Financial Markets^{*}

Zhanhui Chen[†] Jiang Luo[‡] Chongwu Xia[§]

Abstract

We present a theory of endogenous coalition formation in financial markets, which highlights the information sharing and market competition features of coalitions. Allied members enjoy benefits of information advantage and monopolistic power in trading, but forming coalitions incurs direct costs of setting up coalitions and indirect costs from market liquidity dry-ups. Such a trade-off determines the coalition structure of the economy. As allied members behave more monopolistically, coalitions have negative effects on price informativeness and market liquidity. From the information perspective, financial intermediaries (e.g., asset management companies in

[§]Division of Banking & Finance, Nanyang Business School, Nanyang Technological University, Singapore 639798, E-mail: xiac0002@e.ntu.edu.sg.

^{*}We thank Antonio Bernardo, Henry Cao, David Easley, Bing Han, Burton Hollifield, Spencer Martin, Jos van Bommel, Michael Brolley, Wei Xiong, Hongjun Yan, Dongyan Ye, Xiaoyun Yu, Zhuo Zhong, and seminar participants at Fudan University, Nanyang Technological University, University of Melbourne, FIRS 2015, CICF 2015, and FMA 2015 for valuable comments. All errors are ours.

[†]Corresponding author: Zhanhui Chen, Department of Finance, School of Business and Management, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong. Tel.: +852-2358-7670; Fax: +852-2358-1749; E-mail: chenzhanhui@ust.hk.

[‡]Division of Banking & Finance, Nanyang Business School, Nanyang Technological University, 50 Nanyang Avenue S3-01C-75, Singapore 639798. Tel.: +65-6790-4662; Fax: +65-6791-3697; E-mail: luojiang@ntu.edu.sg.

the mutual fund industry) can be viewed as coalitions of of market players (e.g., fund managers). Our theory provides novel insights about the structure of this industry.

JEL Classification: G14; G23; L22

Keywords: Coalition; Financial Intermediaries; Information; Market Competition

1 Introduction

Numerous empirical evidence suggests that networks facilitate information sharing and coordination among market participants.¹ Existing theories on networks in financial markets, including Colla and Mele (2010), Ozsoylev and Walden (2011), Han and Yang (2013), Cao and Ye (2014), and Hollifield et al. (2017) treat these networks as exogenously given and proceed to examine their implications for financial market outcomes. In contrast, working from an information sharing and market competition perspective, we present a theory of endogenous coalition formation decisions by market players to study the coalition structure. We try to address the following questions. What kind of players, for example, well-informed traders or poorly-informed traders, tend to join a coalition? How many investors will join a coalition? As the advance of information technology expedites information processing and dissemination, how does it affect the coalitions in the economy?

A natural framework to study coalition formation in financial markets is a multi-trader generalization of Kyle (1985).² The elements are a risky asset (the stock), multiple in-

¹Networks could be loosely formed by the cultural, educational, religious, or even geographical reasons, or well organized ones like mutual fund managers in the same fund family. For example, Grinblatt and Keloharju (2001) find that investors are more likely to hold, buy, and sell stocks of firms that are located close to them, that communicate in their native tongue, and that have chief executives of the same cultural background. Hong et al. (2004) find that social households-those who interact with their neighbors, or attend church-are more likely to invest in the stock market when their peers participate. Hong et al. (2005) find that mutual fund managers are more likely to buy or sell a particular stock if other managers in the same city are buying or selling that stock, which suggests that investors spread information about stocks to one another by word of mouth. Ivković and Weisbenner (2007) find that have board members of the same education background. Nanda et al. (2004), Gaspar et al. (2006), Elton et al. (2007), and Bhattacharya et al. (2013) show that sibling mutual funds in the same family tend to coordinate trades.

 $^{^{2}}$ Kyle's (1985) setup provides analytical tractability and has been used extensively in the microstructure literature (e.g., Holden and Subrahmanyam, 1992; Foster and Viswanathan, 1996; and Back et al., 2000).

formed traders, a market maker, and a liquidity trader. We modify this framework by allowing informed traders to endogenously choose whether to form a coalition with other informed traders, in which case they will combine resources so that they not only share information but also coordinate trades. The benefit of forming a coalition is that allied members gain information advantage and monopolistic power in trading. However, the coalition has two types of costs. First, there can be direct costs, such as costs related to search, setup, and coordination efforts. Second, there can be an indirect cost. As the coalition gains monopolistic power and behaves more strategically, the market maker lowers market liquidity to break even. This dries up market liquidity. Thus, in our model, whether to form a coalition is an endogenous decision that informed traders make after weighing the benefits and costs. It is possible that all informed traders form a comprehensive coalition. It is also possible that some informed traders form several smaller coalitions, while other informed traders choose to remain independent.

A simple cost-benefit analysis suggests that for a coalition to prevail, it must significantly improve the allied members' information advantage and/or reduce competition. The main results of our model on the coalition structure are consistent with this intuition. First, a coalition is likely to be formed by informed traders with the best-quality information. Second, a coalition, if formed, must be sufficiently large. Third, as the average information quality increases, the coalition size (i.e., the number of informed traders in a coalition) increases and, accordingly, the number of coalitions decreases. Fourth, if there are different groups of informed traders living in separate "islands," so they can form coalitions only within each group, then a small number of large coalitions are likely to emerge from the group of informed traders with good-quality information and the group of informed traders with a large population.³

³This separate "island" setup is often used by the social-network literature (see Jackson, 2010, for a

We further use our model to obtain new insights about the industry structure of financial intermediaries. From the information perspective, financial intermediaries can be viewed as sets of coalitions. Particularly, an asset management company in the mutual fund industry can be viewed as a coalition of several fund managers.⁴ Our model implies that a big asset management company is likely to include fund managers with the best-quality information. Therefore, mutual funds affiliated with big asset management companies tend to outperform those affiliated with small asset management companies.⁵ Moreover, as the advance of information technology expedites information processing and dissemination, which improves the average information quality, in recent decades, industry consolidation has led to the emergence of super big asset management companies such as BlackRock. Finally, big asset management companies tend to arise in metropolitan areas in which there is a large population and investors receive good-quality information. Areas that fit the description include Boston, Chicago, New York, Philadelphia, Los Angeles, and San Francisco.

Our study belongs to the growing literature on networks in financial markets. Existing theories, including Colla and Mele (2010), Ozsoylev and Walden (2011), Han and Yang (2013), Cao and Ye (2014), and Hollifield et al. (2017) assume that informed traders share information through exogenously given networks, and examine the implications of networks for financial market outcomes. We depart from the literature by allowing market players to choose endogenously whether to form coalitions after weighing the benefits and

survey).

⁴Consistent with this view, Bhojraj et al. (2012) show that sibling mutual funds in the same company tend to share information. Nanda et al. (2004), Gaspar et al. (2006), Elton et al. (2007), and Bhattacharya et al. (2013) show that sibling mutual funds in the same company tend to coordinate trades.

Other financial intermediaries can also be viewed as sets of coalitions. For example, a brokerage house can be viewed as a coalition of financial analysts.

 $^{^{5}}$ This is consistent with the empirical findings of Chen et al. (2004), Pollet and Wilson (2008), and Bhojraj et al. (2012).

costs. Our analysis is focused on the coalition structure of the economy.

Our prediction on the implication of information networks for financial market outcomes also differs from those of existing theories. In our model, coalitions increase informed traders' monopolistic power. As they trade more strategically, the informational efficiency of stock prices and market liquidity decrease. In contrast, in Colla and Mele (2010) and Ozsoylev and Walden (2011), information networks lower informed traders' monopolistic power. As informed traders trade more competitively, the informational efficiency of stock prices and market liquidity improve. Han and Yang (2013) show that investors may find it too costly to acquire information by themselves if they can free-ride peers through connections. Thus, networks can lower the total amount of information in the financial markets and, thereafter, the informational efficiency of stock prices. Empirically, their prediction of the negative effect of networks on the informational efficiency of stock prices mostly applies to small stocks, young stocks, and risky stocks, among which information acquisition costs are high.

Our study is also related to the economics literature on strategic network formation.⁶ Existing theories on network formation of financial institutions are predominantly focused on the banking sector.⁷ Little is known about network formation in other sectors such as the mutual fund industry. Our study fills the void. We argue that asset management companies in the mutual fund industry can be viewed as coalitions endogenously chosen by fund managers in pursuit of information advantage and monopolistic power in trading. This intrepretation fits some important aspects of this industry. Our model provides novel insights about the structure of this industry.

⁶For theories in general settings, see, for example, Jackson and Wolinsky (1996), Bala and Goyal (2000), and Hojman and Szeidl (2006, 2008). Jackson (2010), and Allen and Babus (2009) provide excellent surveys.

⁷See, for example, Blume et al. (2011), Allen et al. (2012), Cabrales et al. (2012), Acemoglu et al. (2013), Babus (2013), Zawadowski (2013), Bluhm et al. (2013), and Farboodi (2015).

Existing theories on delegated asset management, including Ou-Yang (2003), Palomino and Prat (2003), Gervais et al. (2005), Dybvig et al. (2010), Vayanos and Woolley (2013), and He and Xiong (2013), are focused on the design of the optimal contracts to incentivize fund managers to alleviate the agency problem at the fund level. They typically assume that fund managers work for the asset management companies, the structure of which is given exogenously.

We organize the rest of the article as follows. Section 2 describes our model. Section 3 solves for the equilibrium and then derives its properties. Section 4 applies the predictions of our model to study the industry structure of financial intermediaries. Section 5 concludes.

2 The Model

We develop our model based on a multi-trader generalization of Kyle (1985). Consider a simple exchange economy. There are three dates, -1, 0, and 1. There is one risky asset, a stock. The stock pays $\overline{F} + X$ at date 1. \overline{F} is a positive constant. X is a random variable that follows a standard normal distribution; that is, $X \sim N(0, 1)$. The stock price at date 0 is denoted as P, which is to be determined.

There are three types of risk-neutral players. First, there are J informed traders. At period 0, each informed trader, indexed by j, observes a private signal about the payoff of the stock, $S_j = X + \epsilon_j$. ϵ_j follows a normal distribution with mean zero and variance v_j ; that is, $\epsilon_j \sim N(0, v_j)$. She submits a market order, D_j , which is to be determined. We assume that $\forall i \neq j$, ϵ_i is independent of ϵ_j .

Second, there is a liquidity trader. She submits a market order, Z, which follows a normal distribution with mean zero and variance σ_z^2 ; that is, $Z \sim N(0, \sigma_z^2)$. We assume that Z is independent of X and ϵ_j , $\forall j$. Third, there is a market maker. She receives the aggregate market order, $D = \sum_j D_j + Z$, and sets the stock price, P. The market maker faces perfect competition from other market makers, so she has zero expected profits. This implies that $P = \bar{F} + E[X|D]$.

The novel feature of our model is that at date -1, informed traders decide whether to ally with peers. If they do, then they not only exchange information truthfully but also coordinate trades. A coalition, which does not necessarily include all informed traders, is indicated by a set A.

Here we assume that within a coalition, informed traders will tell each other their information truthfully. There are various mechanisms to ensure this. Take the coalition formed among neighbors or alumni as an example. There can be a severe reputation loss if a member is caught in a lie. It is also possible that allied members' payoffs are aligned, so they have no incentives to lie. This tends to happen among professional money managers in the same asset management company because their compensations are often partially linked to company performance. Finally, some institutional features might prevent such deviation activities. For example, the SEC prohibits cross trades between sibling funds in the same family that may cause conflicts of interests.

Coordination of trades is a key ingredient of our model. Stein (2008) points out that in general, if informed traders cannot coordinate trades, then they will not exchange information either. The intuition for this is that information exchange will lower their information advantage. What's more, receivers of this information may even use it when trading against those who provided it. Colla and Mele (2010) explain this intuition in more details. They argue that private information gives informed traders monopolistic power. If informed traders exchange information but don't coordinate trades, then their monopolistic power weakens, so they will trade more aggressively to preempt peers. It can be shown that their payoff will decrease. Therefore, they will not exchange information in the first place.

Forming a coalition, A, causes direct costs, C(A), due to search, setup, and coordination efforts. In general, C(A) increases with the extent and complexity of the coalition, A, which can be measured using the number of informed traders in the coalition. Allied members in the coalition share the costs and benefits of the coalition based on the Shapley (1953) value, which captures their contributions to the coalition. In our model, an allied member's Shapley value is proportional to her information precision, $1/v_i$.

We summarize the timeline of our model as follows.

- Date -1: (Coalition Formation Stage) Informed traders decide whether to form a coalition, A, which does not necessarily include all informed traders.
- Date 0: (Trading Stage) Each informed trader, j, receives her private information, S_j . Informed traders in the coalition, $j \in A$, share their private information, S_j , and coordinate trades. The coalition submits a market order, D_A . Each independent informed trader, $j \notin A$, submits a market order, D_j , based on her private information, S_j . The liquidity trader submits a market order, Z. After receiving the aggregate market order, $D = D_A + \sum_{j \notin A} D_j + Z$, the market maker sets the stock price, P.
- Date 1: (Final Stage) The stock pays $\overline{F} + X$, which is distributed to every player according to her holdings.

In our model, whether to form a coalition so that they can exchange information and coordinate trades is an endogenous decision that informed traders make after weighing the benefits and costs. It is possible that all informed traders form a comprehensive coalition. It is also possible that some informed traders form several smaller coalitions, while other informed traders choose to remain independent.

3 The Equilibrium

We solve for the equilibrium using backward induction. In Section 3.1, we focus on the trading stage at date 0. We suppose that a coalition, A, has already been formed and solve for every player's optimal trading strategy. Our analysis can easily be extended to include multiple coalitions. In Section 3.2, we focus on the coalition formation stage at date -1, where we study what kind of informed traders form a coalition, how big a coalition can be, and how many coalitions can be formed.

3.1 Trading Stage

3.1.1 Trading with No Coalition

Before jumping to the case with coalitions, we first study a benchmark case in which there is no coalition, so all informed investors compete against each other.

Proposition 1. Consider the case with no coalition of informed traders, which is denoted by (NA). There is a linear equilibrium in which the market maker sets the stock price according to $P(NA) = \overline{F} + \lambda(NA)D$, and each informed trader submits a market order $D_j(NA) = \theta_j(NA)S_j$. $\lambda(NA)$ and $\theta_j(NA)$ are given by:

$$\lambda(NA) = \frac{K(NA)/\sigma_z}{1 + \sum_j \frac{1}{1 + 2v_j}},$$

$$\theta_j(NA) = \frac{\sigma_z/K(NA)}{1 + 2v_j},$$

where $K(NA) = \sqrt{\sum_{j} \frac{1 + v_j}{(1 + 2v_j)^2}}.$

Proof: See the Appendix.

To understand this equilibrium, we consider the following special case, which Holden and Subrahmanyam (1992) have studied. The intuitions we obtain in this special case help us understand the equilibrium when there are coalitions of informed traders.

<u>A special case</u>: Let $v_j = V$, $\forall j$, so every informed trader has the same-quality information. Denote the informativeness of the stock price as Var(X|P). It is straightforward to show that

$$\operatorname{Var}(X|P(NA)) = \frac{1}{1+J/(1+2V)},$$

$$\theta_j(NA) = \frac{\sigma_z}{\sqrt{J(1+V)}},$$

$$\lambda(NA) = \frac{\sqrt{J(1+V)}/\sigma_z}{1+2V+J}.$$

A decrease in J has three consequences. First, the informativeness of the stock price decreases (high Var(X|P)). There are two reasons for this. One reason is that as J decreases, the sources of information decrease, which lowers the total amount of information in the market. The other reason is that there is less competition among informed traders, so they behave more monopolistically. Their trades reveal less information to the market. Second, informed traders trade more aggressively (high θ_j). This is because as the competition decreases, informed traders are less subject to the winner's curse problem. Third, market liquidity decreases (high λ) for sufficiently large J, such as J > 1 + 2V. This is because as informed traders behave more monopolistically, the market maker must increase λ to break even.

An increase in informed traders' information quality (i.e., V decreases) also has three consequences. First, the price informativeness increases (low Var(X|P)). This is because informed traders reveal more precise information through trades to the stock price. Second, informed traders trade more aggressively (high θ_j). This is because they are more confident in their own information. Third, market liquidity decreases (high λ) if informed traders' information is not very precise, such as 3 + 2V > J. This is because the market maker faces a more severe information disadvantage and must increase λ to break even.

3.1.2 Trading with a Coalition

Suppose that a coalition of some informed traders, A, has been formed. Denote

$$S_A = \frac{\sum_{j \in A} \frac{S_j}{v_j}}{\sum_{j \in A} \frac{1}{v_j}} = X + \epsilon_A.$$

 $\epsilon_A = \frac{\sum_{j \in A} \frac{\epsilon_j}{v_j}}{\sum_{j \in A} \frac{1}{v_j}} \text{ follows a normal distribution with mean zero and variance } v_A = 1 / \sum_{j \in A} \frac{1}{v_j};$ that is, $\epsilon_A \sim N(0, v_A).$

Lemma 1. As far as X is concerned, S_A is a sufficient statistic of $\{S_j : j \in A\}$.

Proof: See the Appendix.

An implication of this lemma is that we can treat the coalition, A, as one informed trader who observes a private signal S_A . In the following proposition, we use this idea to describe the coalition's demand for the stock.

Proposition 2. Consider the case with a coalition of informed traders, which is denoted by (A). There is a linear equilibrium in which the market maker sets the stock price according to $P(A) = \overline{F} + \lambda(A)D$, the coalition submits a market order $D_A(A) = \theta_A(A)S_A$, and every other informed trader, $j \notin A$, submits a market order $D_j(A) = \theta_j(A)S_j$. $\lambda(A)$, $\theta_A(A)$, and $\theta_j(A)$ are given by:

$$\begin{split} \lambda(A) &= \frac{K(A)/\sigma_z}{1 + \frac{1}{1 + 2v_A} + \sum_{j \notin A} \frac{1}{1 + 2v_j}}, \\ \theta_A(A) &= \frac{\sigma_z/K(A)}{1 + 2v_A}, \\ \theta_j(A) &= \frac{\sigma_z/K(A)}{1 + 2v_j}, \, \forall j \notin A, \end{split}$$

where
$$K(A) = \sqrt{\frac{1+v_A}{(1+2v_A)^2} + \sum_{j \notin A} \frac{1+v_j}{(1+2v_j)^2}}.$$

Proof: See the Appendix.

The following corollary describes the monotonic properties of the equilibrium with respect to the coalition size.

Corollary 1. Suppose that the coalition expands from A to A', where $A' = A \cup \{b\}$ and $b \notin A$. Then,

- (i) the stock price becomes less informative; that is, $\operatorname{Var}(X|P(A')) > \operatorname{Var}(X|P(A))$;
- (ii) market liquidity decreases; that is, $\lambda(A') > \lambda(A)$;
- (iii) informed traders bid more aggressively; that is, $\theta_{A'}(A') > \theta_A(A)$ and $\theta_j(A') > \theta_j(A)$, $\forall j \notin A'$.

Proof: See the Appendix.

Intuitively, the expansion of the coalition reduces the effective number of informed traders. Then, as the special case in the last subsection suggests, this reduces the intensity

of competition among informed traders. As informed traders both inside and outside the coalition behave more monopolistically, they trade more aggressively but release less information to the market. Therefore, the stock price becomes less informative. The market maker must increase λ , the sensitivity of the stock price to the market order she receives, to break even. This lowers market liquidity.

Corollary 1 suggests that networks, in the form of coalitions, in the financial markets can have negative effects on the informativeness of stock prices and on market liquidity. This market liquidity dry-up represents an indirect but important cost of forming a coalition.

3.2 Coalition Formation Stage

To study the coalition formation, we need to look at the effects of a coalition of informed traders, A, on each player's ex ante (date -1) payoff. An immediate observation is that the coalition has no effect on the market maker's ex ante payoff because on average, her payoff is always 0.

For other players, in the case with no coalition of informed traders, denote $E\Pi_j(NA)$ and $E\Pi_L(NA)$ as the ex ante payoffs to each informed trader and the liquidity trader. It follows from the proof of Proposition 1 that

$$E\Pi_j(NA) = E\left[E\left[(\bar{F} + X - P(NA)) \cdot D_j(NA) \middle| S_j\right]\right] = \lambda(NA)\theta_j^2(NA)(1+v_j),$$

$$E\Pi_L(NA) = E\left[(\bar{F} + X - P(NA)) \cdot Z\right] = -\lambda(NA)\sigma_z^2.$$

In the case with a coalition of informed traders, A, denote $E\Pi_A(A)$, $E\Pi_j(A)$ ($\forall j \notin A$), and $E\Pi_L(A)$ as the ex ante payoffs to the coalition, each independent informed trader, and the liquidity trader. It is straightforward to show that

$$E\Pi_{A}(A) = E\left[E\left[(\bar{F} + X - P(A)) \cdot D_{A}(A) \middle| S_{A}\right]\right] = \lambda(A)\theta_{A}^{2}(A)(1 + v_{A}),$$

$$E\Pi_{j}(A) = E\left[E\left[(\bar{F} + X - P(A)) \cdot D_{j}(A) \middle| S_{j}\right]\right] = \lambda(A)\theta_{j}^{2}(A)(1 + v_{j}), \forall j \notin A,$$

$$E\Pi_{L}(A) = E\left[(\bar{F} + X - P(A)) \cdot Z\right] = -\lambda(A)\sigma_{z}^{2}.$$

Use $\Gamma_A(A)$, $\Gamma_j(A)$ ($\forall j \notin A$), and $\Gamma_L(A)$ to describe the effects of the coalition on the ex ante payoffs to the coalition, each independent informed trader, and the liquidity trader.

$$\Gamma_A(A) = E\Pi_A(A) - \sum_{j \in A} E\Pi_j(NA),$$

$$\Gamma_j(A) = E\Pi_j(A) - E\Pi_j(NA), \quad \forall j \notin A,$$

$$\Gamma_L(A) = E\Pi_L(A) - E\Pi_L(NA).$$

Coalition members share the coalition synergy, based on the Shapley value:

$$\Gamma_j(A) = \frac{\frac{1}{v_j}}{\sum_{j \in A} \frac{1}{v_j}} \Gamma_A(A), \quad \forall j \in A.$$

Corollary 2. (i) The coalition increases the ex ante payoff to independent informed traders (if any); that is, $\Gamma_j(A) > 0$, $j \notin A$.

(ii) The coalition decreases the ex ante payoff to the liquidity trader; that is, $\Gamma_{\rm L}(A) < 0$.

Proof: See the Appendix.

A coalition has three effects on an independent informed trader's ex ante payoff. First, it reduces competition among informed traders, which increases her payoff. Second, it represents a stronger competitor, which decreases her payoff. Third, it leads to dry-ups of market liquidity, which also decreases her payoff. Part (i) of Corollary 2 suggests that the first effect dominates the other two effects. Thus, the net effect of a coalition on an independent informed trader's payoff is positive.

Part (ii) of Corollary 2 suggests that a coalition has a negative effect on the liquidity trader's payoff. This is because the coalition causes dry-ups of market liquidity.

Does a coalition increase the ex ante payoff to allied members in the coalition? The answer to this question is not obvious. On one hand, a coalition reduces competition and improves the allied members' information advantage (relative to that of independent informed traders), which increases their payoff. On the other hand, a coalition also causes direct costs due to search, setup, and coordination efforts, which are represented by C(A), and an indirect cost from dry-ups of market liquidity, which decreases their payoff. We are not able to obtain a clear comparison between the benefits and costs of form coalitions except in the following two polar cases regarding C(A).

<u>**Two Polar Cases:**</u> In the first polar case, it is prohibitively costly to form a coalition (i.e., $C(A) \to \infty, \forall A$). In this case, any possible gains from the coalition will be outweighed by the high costs; that is, $\Gamma_A(A) \leq C(A)$. Therefore, no coalition of informed traders will be formed.

In the second polar case, the cost to form a coalition is negligible (i.e., $C(A) \rightarrow 0$, $\forall A$). In this case, a coalition that includes all informed traders will be formed. We refer to this coalition as a comprehensive coalition. One may wonder whether an informed trader prefers to leave the coalition and remain independent. We can show that if all other informed traders stay in the coalition, then she will also stay in the coalition. Therefore, a comprehensive coalition is sustained as a Nash equilibrium.

In what follows, we assume that C(A) is prohibitively high for a comprehensive coali-

tion, but reasonably low for a partial coalition. This allows us to focus on the partial coalition(s). We use a numerical analysis to examine under what conditions a partial coalition can improve the payoff of allied members, so they will form a partial coalition.⁸ We consider cases with different levels of information quality, V, and/or different numbers of informed traders, J. This allows us to examine the impacts of information and competition on our results.

3.2.1 Who Joins a Partial Coalition?

Consider a simple economy in which there are only J = 3 informed traders. Let $v_1 \le v_2 \le v_3$, so they are ordered by their information quality. Suppose that the cost function, C(A), satisfies mild conditions, which allows only a partial coalition of two informed traders. Who will join the coalition?

There can be three scenarios for the coalition structure.

- (S1) $\{1, 2, 3\}$ indicates that every informed trader remains independent.
- (S2) $\{\{1,2\},3\}$ indicates that informed traders 1 and 2 form a coalition.
- (S3) $\{\{1,3\},2\}$ indicates that informed traders 1 and 3 form a coalition.
- (S4) $\{1, \{2, 3\}\}$ indicates that informed traders 2 and 3 form a coalition.

In general, there can be multiple equilibria for a game with three or more players. To simplify our analysis, we follow Jackson and Wolinsky (1996) to give two definitions about the equilibrium.⁹

⁸We use numerical analysis to illustrate the coalition structure of the economy because we are not able to obtain the analytical solution to our model. We have varied our numerical analysis to verify the robustness of our main results.

⁹Farboodi (2014) uses a similar approach to define the network equilibrium in the banking sector.

Definition 1. A coalition, A, is admissible if it provides a positive synergy, $\Gamma(A) > 0$.

Definition 2. A coalition, A, is stable if no player in the coalition prefers to leave.

A simple cost-benefit analysis suggests that for a partial coalition to prevail, it must significantly improve the allied members' information advantage and/or reduce competition. In this simple economy with three informed traders, all partial coalitions include two informed traders, so they reduce competition to the same degree. Therefore, the prevailing partial coalition hinges solely on its information advantage. This implies that the partial coalition should include informed traders with the best-quality information. Informed traders with the poorest-quality information should remain independent. The following corollary is consistent with this intuition.

Corollary 3. Consider a simple economy with J = 3 and $v_1 \le v_2 \le v_3$.

- (i) A coalition between information traders 2 and 3 is not admissible because it produces a negative synergy; that is, $\Gamma_A(A = \{2, 3\})) \leq 0.$
- (ii) A coalition between informed traders 1 and 2 produces a higher synergy than a coalition between informed traders 1 and 3; that is, $\Gamma_A(A = \{1, 2\}) \ge \Gamma_A(A = \{1, 3\}).$

(iii) Informed trader 1 receives more benefit from a coalition with informed trader 2 than from a coalition with informed trader 3; that is, $\Gamma_1(A = \{1, 2\}) \equiv \frac{1/v_1}{1/v_1 + 1/v_2} \Gamma_A(A = \{1, 2\}) \geq \Gamma_1(A = \{1, 3\}) \equiv \frac{1/v_1}{1/v_1 + 1/v_3} \Gamma_A(A = \{1, 3\}).$

Proof: See the Appendix.

Part (i) of Corollary 3 rules out the partial coalition of informed traders 2 and 3 because this coalition produces a negative synergy. Parts (ii) and (iii) suggest that the partial coalition, $A = \{1, 2\}$, is likely to prevail not only because it produces a higher

synergy but also because it is preferred by informed trader 1 as she gains more from the synergy of this coalition, according to the Shapley value. For it to be admissible, it must be

$$\Gamma_A(A = \{1, 2\}) \ge \max(0, \Gamma_A(A = \{1, 3\}), \Gamma_A(A = \{2, 3\})).$$
(1)

Figure 1 depicts the admissible coalition, $A = \{1, 2\}$, using a numerical analysis. We let $1 = v_1 \leq v_2 \leq v_3 \leq 40$. In the shaded area, Eq. (1) holds, so the partial coalition, $A = \{1, 2\}$, is admissible. Note that this area features a high value of v_3 (≥ 3.5). Therefore, a partial coalition can prevail only if the independent informed traders do not have very precise information. This also confirms our intuition that a partial coalition increases the payoff to the allied informed players when it significantly improves the allied informed traders' information advantage.

[Insert Figure 1 here.]

Part (iii) of Corollary 3 indicates that informed trader 1 prefers to ally with informed trader 2. For the partial coalition, $A = \{1, 2\}$, to be stable, informed trader 2 should have no incentive to leave the coalition. Jackson and Wolinsky (1996) suggest a condition in addition to Eq. (1).

$$\Gamma_2(A = \{1, 2\}) \equiv \frac{1/v_2}{1/v_1 + 1/v_2} \Gamma_A(A = \{1, 2\}) \ge \Gamma_2(A = \{1, 3\}).$$
(2)

This equation ensures that informed trader 2 gains more from staying in the coalition than from becoming independent while letting informed traders 1 and 3 form a coalition.

[Insert Figure 2 here.]

Figure 2 shows that in the dark-shaded area, the coalition, $A = \{1, 2\}$, is stable. In the light-shaded area, the coalition, $A = \{1, 2\}$, is fragile, so informed trader 2 may leave the coalition. There are two notable observations. First, the stable area features a higher v_3 than the fragile area does. This suggests that to be stable, a coalition needs to have a significant information advantage (relative to informed trader 3). Second, the stable area also features a much smaller v_2 . This suggests that to be stable, the two informed traders, 1 and 2, in the coalition must have similar-quality information, so they have a similar share of the coalition synergy.

[Insert Figure 3 here.]

In what follows, we restrict our attention to the admissibility of the partial coalition, $A = \{1, 2\}$. Figure 3 considers a more general case. There are J = 10 informed traders. Their information qualities satisfy $1 = v_1 \le v_2 \le v_3 \le 20$ and $v_j = 50$, $\forall j > 3$. The shaded area, which features a higher value of v_3 (≥ 6.5), satisfies Eq. (1), so the coalition $A = \{1, 2\}$ is admissible in this area. An interesting observation from comparing Figures 3 and 1 is that as J increases, independent informed trader 3 must have poorer information quality for the partial coalition $A = \{1, 2\}$ to be admissible. This is because as there are more independent informed traders, independent informed trader 3 must have poorer information quality, so that the coalition $A = \{1, 2\}$ can maintain sufficient information advantage to make a profit.

[Insert Figure 4 here.]

In Figure 4, we let $v_j = 35$, $\forall j > 3$, so these informed traders have relatively better information quality than those in Figure 3. Compared with Figure 3, the shaded area, in which the coalition $A = \{1, 2\}$ is admissible, shifts farther upwards. This area features an even higher value of $v_3 (\geq 9.5)$. This is because as independent informed traders j > 3 have better information quality, independent informed trader 3 must have even poorer information quality, so the coalition $A = \{1, 2\}$ can maintain sufficient information advantage to make a profit.

Prediction 1. A partial coalition tends to be formed by informed traders with the bestquality information.

3.2.2 Minimum Coalition Size

Consider a symmetric case in which $v_j = V$, $\forall j$, so all informed traders have the samequality information. Figures 1 to 4 suggest that in this symmetric case, a small partial coalition $A = \{1, 2\}$ won't be formed because it does not significantly reduce competition and gain information advantage. Figure 5 confirms this. Suppose that there can be only one coalition. Panel (a) shows that if formed, the partial coalition $A = \{1, 2\}$ will have a negative synergy (i.e., $\Gamma_A(A = \{1, 2\}) < 0$). Panel (b) shows that consistent with Corollary 2, independent informed traders generally gain from this coalition (i.e., $\Gamma_j(A = \{1, 2\}) > 0$, $\forall j \geq 3$).

[Insert Figure 5 here.]

Our above cost-benefit analysis suggests that for a partial coalition to be admissible, it needs to be sufficiently large because only a large coalition can significantly reduce competition and obtain information advantage. The minimum size of an admissible coalition, \underline{m} , can be obtained as follows:

$$\min_{m} \quad m \\ \text{s.t.} \quad \Gamma_{A}(A = \{1, 2, ..., m\}) \ge 0.$$

Figure 6 plots \underline{m} depending on the information quality. We assume that there are J = 40 informed traders, each of whom has the same-quality information, $v_j = V, \forall j$.

Here we allow for multiple coalitions. Because of symmetry, every coalition should have the same number of informed players. There are two notable observations. First, \underline{m} increases with the information quality, V. For example, \underline{m} for a single coalition is 27 when V = 10, and it increases to 30 when V = 5. This is because as other informed traders have good-quality information, the coalition must be sufficiently large to reduce competition and gain information advantage. Second, \underline{m} decreases with the number of coalitions. This suggests that coalitions can have an externality effect on one another because each coalition lowers the intensity of competition in the market, benefiting other coalitions. This lowers the minimum size of a coalition.

[Insert Figure 6 here.]

Figure 7 plots \underline{m} depending on the number of coalitions. We assume that there can be J = 20, 30, and 40 informed traders. Consistent with Figure 6, \underline{m} decreases in the number of coalitions. Moreover, \underline{m} increases in the total number of informed traders. This is because when there are many informed traders, a coalition must be sufficiently large to reduce competition and gain information advantage.

[Insert Figure 7 here.]

Prediction 2. A coalition, if formed, must be sufficiently large.

3.2.3 Optimal Coalition Structure

Now we study the optimal coalition structure. To simplify our analysis, we let $v_j = V, \forall j$, so all informed traders have the same-quality information. We follow Dessein and Santos (2006) to assume a quadratic cost function for a coalition:

$$C(A(m)) = \gamma(m-1)^2,$$

where m represents the number of allied informed traders in the coalition. γ is a constant.

By symmetry, there can be n coalitions, each of which has m informed traders.¹⁰ Allied members have the same Shapley value, so they share the coalition synergy equally. We define the optimal coalition structure of the economy as follows.

Definition 3. Consider a symmetric economy, in which $v_j = V$, $\forall j$, so all informed traders have the same-quality information. A coalition structure of the economy, $\{m, n\}$, is optimal if it maximizes the average coalition member's net gain, that is,

$$\max_{m,n} \quad \frac{1}{m} \left[\Gamma_A(A = \{1, 2, ..., m\}) - C(A(m)) \right].$$

This optimal coalition structure, $\{m, n\}$, can sustain as an equilibrium if any allied player's deviation from this structure leads to the collapse of the structure and the only offequilibrium outcome is that every informed player stays independent. Admittedly, there can be other equilibria. We shy away from the other equilibria for two reasons. First, there is little tractability of these equilibria. Second, from the perspectives of informed traders, these equilibria are obviously *Pareto* dominated by the optimal coalition structure we consider here.

In Figure 8, we assume that there can be J = 30 or 40 informed traders. We let $C(A(m)) = 10^{-4}(m-1)^2$. Panels (a) and (b) show that as the information quality improves (low V), the optimal coalition size, m, increases, and the optimal number of coalitions, n, decreases. The intuition for these results is as follows. As informed traders have good-quality information, only large coalitions can significantly reduce competition and gain information advantage, leading to a profit. Therefore, the optimal coalition size should

¹⁰If the total number of informed traders, J, is not divisible by n, then there will be some residual informed traders. We assume that they remain independent.

increase. This also leads to a smaller number of coalitions because in this simple example, the population of informed traders is constant.

Panels (a) and (b) also show that as the number of informed traders increases (high J), the optimal coalition size may or may not increase, but the number of coalitions generally increases. Intuitively, a larger number of coalitions implies that the competition is significantly reduced, so the coalitions can make a profit.

[Insert Figure 8 here.]

Prediction 3. As information quality improves, the optimal coalition size increases and the number of coalitions decreases.

Next, we consider the case in which there are two groups of informed traders, indexed by the subscripts 1 and 2, in the economy. Informed traders from the same group have the same-quality information. Specifically, if informed trader j is in group 1 (group 2), then $v_j = V_1$ ($v_j = V_2$). We follow the social-network literature (e.g., Jackson, 2008) to assume that the two groups of informed traders live in two separate "islands," so they can form coalitions only within each group. In our framework, this requires that the costs of maintaining an across-group coalition, which can be related to monitoring and enforcing, be prohibitively high.

By symmetry, in group 1 (2), there can be n_1 (n_2) coalitions, each of which has m_1 (m_2) informed traders. In a similar vein as our Definition 3 on optimality (see above), we assume that the two groups are equally weighted in the sense that the optimal coalition structure m_1, m_2, n_1, n_2 maximizes the net gains of the average coalition member from each group:

$$\max_{m_1, m_2, n_1, n_2} \frac{1}{m_1} \Big[\Gamma_A(A = \{1, ..., m_1\}) - C(A(m_1)) \Big] + \frac{1}{m_2} \Big[\Gamma_A(A = \{1, ..., m_2\}) - C(A(m_2)) \Big].$$

[Insert Figure 9 here.]

In Figure 9, we assume that each group has 40 informed traders. Informed traders in group 1 have better-quality information than those in group 2; specifically, $V_1 = 1$ and $V_2 > 1$. We also let $C(A(m)) = 10^{-4}(m-1)^2$. Panels (a) and (b) show that group 1 has a larger optimal coalition size but a smaller number of coalitions than group 2 does. The intuition for these results is as follows. As informed traders in group 1 have relatively good-quality information, only large coalitions can significantly reduce competition and gain information advantage, leading to a profit. Therefore, group 1 should have a larger coalition size, which also leads to a smaller number of coalitions. By significantly reducing competition in the whole economy, group 1's coalition structure, large m and small n, also has a spillover effect on group 2's coalition structure. Group 2's optimal coalition size doesn't need to be very large, but the coalitions can still make a profit.

[Insert Figure 10 here.]

In Figure 10, we assume that there are more informed traders in group 1 than in group 2; that is, $J_1 > 100$ and $J_2 = 100$. We let all informed traders have the same-quality information, $V_1 = V_2 = 5$. Also, $C(A(m)) = 10^{-5}(m-1)^2$. Panels (a) and (b) show that group 1 has a large optimal coalition size but a smaller number of coalitions than group 2 does.¹¹ The intuition for these results is similar to that for the case in Figure 9. As there are more informed traders in group 1, only large coalitions can significantly reduce competition and gain information advantage, leading to a profit. Therefore, group 1 should have a larger coalition size, which also leads to a smaller number of coalitions. By significantly reducing competition in the whole economy, group 1's coalition structure, large m and small n, also has a spillover effect on group 2's coalition structure. Group

¹¹Note that there are some bumps in the plots, due to the nature of integer programming.

2's optimal coalition size doesn't need to be very large, but the coalitions can still make a profit.

Prediction 4. A small number of large coalitions are likely to originate from the group of informed traders who possess good-quality information and the group of informed traders with a large population.

4 Understanding the Industry Structure of Financial Intermediaries

In this section, we use our theory to obtain new insights about the industry structure of financial intermediaries. We are particularly interested in the mutual fund industry because after experiencing rapid growth in the past decades, this industry has become one of the most important financial intermediaries. Toward the end of 2014, the total assets under management by the U.S. mutual fund industry reached \$15.9 trillion, with 43.3% of households investing in mutual funds (Investment Company Institute, 2015). Yet, there is little understanding about the structure of this industry.

From the information perspective, an asset management company in the mutual fund industry can be interpreted as a coalition of professional money managers. This view fits some important aspects of this industry (see the citations in Footnote 4). Our theory has the following implications.

4.1 Firm Size and Performance

Predictions 1 and 2 imply that a big asset management company is likely to include fund managers with the best-quality information. This implication is consistent with empirical findings by Chen et al. (2004) and Pollet and Wilson (2008). They show that mutual funds affiliated with big asset management companies tend to outperform those affiliated with small asset management companies. Bhojraj et al. (2012) further show that mutual fund managers affiliated with big asset management companies indeed possess an information advantage over those affiliated with small asset management companies.

4.2 Technology and Firm Size

Prediction 3 implies that as information environment improves, the consolidation of the mutual fund industry will lead to the emergence of a few big asset management companies. Recent decades have seen fast growth in information technology, which improves information quality. During the same period, the mutual fund industry experienced a wave of mergers, which gave rise to a few gigantic firms. For example, BlackRock acquired State Street Research in 2005, Merrill Lynch Investment Managers in 2006, Quellos Group in 2007, and Barclays Global Investors in 2009. As of September 2014, Blackrock is the biggest asset management company.

4.3 The Geography

Prediction 4 implies that the mutual fund industry can have an interesting geographic patterns. Specifically, a small number of big asset management companies tend to arise in areas in which there is a large population and investors receive good-quality information. The areas in the U.S. that fit this description include Boston, Chicago, New York, Philadelphia, Los Angeles, and San Francisco.

5 Conclusions

In this article, we develop a theory of endogenous coalition formation decisions by market players to study the coalition structure in financial markets. In our model, the benefit of forming a coalition is that the allied members gain information advantage as well as monopolistic power in trading. However, the coalition has two types of costs. First, there can be direct costs, such as costs related to search, setup, and coordination efforts. Second, there can be an indirect cost. Specifically, as the coalition gains monopolistic power and behaves more strategically, the market maker lowers market liquidity to break even. This dries up market liquidity. Such a trade-off determines the coalition structure in the economy.

Our model has several interesting results on the coalition structure. First, a coalition is likely to be formed by informed traders with the best-quality information. Second, a coalition, if formed, must be sufficiently large. Third, as the advance of information technology expedites information processing and dissemination, the coalition size increases and, accordingly, the number of coalitions decreases. Fourth, a small number of large coalitions are likely to emerge from the group of informed traders with good-quality information and the group of informed traders with a large population.

From the information perspective, financial intermediaries (e.g., asset management companies and brokerage houses) can be viewed as sets of coalitions endogenously chosen by market players (e.g., fund managers and financial analysts) in pursuit of information advantage and monopolistic power in trading. Our model provides novel insights about the industry structure of financial intermediaries.

Our model also predicts that networks, in the form of coalitions, have negative effects on the informativeness of stock prices and market liquidity. This prediction is different from predictions of existing theories in the literature, such as Colla and Mele (2010) and Ozsoylev and Walden (2011).

References

- Acemoglu, D., A. Ozdaglar, and A. Tahbaz-Salehi, 2013, Systemic risk and stability in financial networks, Working Paper.
- [2] Allen, F., and A. Babus, 2009, Networks in finance, in P. Kleindorfer and J. Wind (eds.), Network-based Strategies and Competencies, 367-382.
- [3] Allen, F., A. Babus, and E. Carletti, 2012, Asset commonality, debt maturity and systemic risk, Journal of Financial Economics 104, 519-534.
- [4] Babus, A., 2013, The formation of financial networks, Working Paper.
- [5] Back, K., H. H. Cao, and G. A. Willard, 2000, Imperfect competition among informed traders, Journal of Finance 55, 2117-2155.
- [6] Bala, V., and S. Goyal, 2000, A noncooperative model of network formation. Econometrica 68, 1181-1229.

Insiders, gurus, and credibility, Quarterly Journal of Economics 107, 921-958.

- [7] Bhattacharya, U., J. H. Lee, and V. K. Pool, 2013, Conflicting family values in mutual fund families, Journal of Finance 68, 173-200.
- [8] Bhojraj, S., Y. J. Cho, and N. Yehuda, 2012, Mutual fund family size and mutual fund performance: The role of regulatory changes, Journal of Accounting Research 50, 647-684.
- [9] Bluhm, M., E. Faia, and J. P. Krahnen, 2013, Endogenous banks€? network, cascades and systemic risk, Working Paper.

- [10] Blume, L., D. Easley, J. Kleinberg, R. Kleinberg, and E. Tardos, 2011, Network formation in the presence of contagious risk, in Proceedings of the 12th ACM conference on Electronic commerce, 1-10.
- [11] Cabrales, A., P. Gottardi, and F. Vega-Redondo, 2012, Risk-sharing and contagion in networks, Working Paper.
- [12] Cao, H. H. and D. Ye, 2014, Social network, herding and competition, Working Paper.
- [13] Chen, J., H. Hong, M. Huang, and J. D. Kubik, 2004, Does fund size erode mutual fund performance? The role of liquidity and organization, American Economic Review 90, 1276-1302.
- [14] Cohen, L., A. Frazzini, and C. Malloy, 2008, The small world of investing: Board connections and mutual fund returns, Journal of Political Economy 116, 951-979.
- [15] Colla, P., and A. Mele, 2010, Information linkages and correlated trading, Review of Financial Studies 23, 203-246.
- [16] Dessein, W., and T. Santos, 2006, Adaptive organizations, Journal of Political Economy 114, 956-995.
- [17] Dybvig, P., H. Farnsworth, and J. Carpenter, 2010, Portfolio performance and agency, Review of Financial Studies 23, 1-23.
- [18] Elton, E. J., M. J. Gruber, and T. C. Green, 2007, The impact of mutual fund family membership on investor risk, Journal of Financial and Quantitative Analysis 42, 257-277.
- [19] Farboodi, M., 2015, Intermediation and voluntary exposure to counterparty risk, Working Paper.

- [20] Foster, F. D., and S. Viswanathan, 1996, Strategic trading when agents forecast the forecasts of others, Journal of Finance 51, 1437-1478.
- [21] Gaspar, J., M. Massa, and P. Matos, 2006, Favoritism in mutual fund families? Evidence on strategic cross-fund subsidization, Journal of Finance 61, 73-104.
- [22] Grinblatt, M., and M. Keloharju, 2001, How distance, language, and culture influence stockholdings and trades, Journal of Finance 56, 1053-1073.
- [23] Gervais, S., A. Lynch, and D. Musto, 2005, Fund families as delegated monitors of money managers, Review of Financial Studies 18, 1139-1169.
- [24] Han, B., and L. Yang, 2013, Social networks, information acquisition, and asset prices, Management Science 59, 1444-1457.
- [25] Hollifield, B., Neklyudov, A., Spatt, C., 2017, Bid-ask spreads, trading networks, and the pricing of securitizations, Review of Financial Studies 30, 3048-3085.
- [26] He, Z., and W. Xiong, 2013, Delegated asset management, investment mandates, and capital immobility, Journal of Financial Economics 107, 239-258.
- [27] Hojman, D. A., and A. Szeidl, 2006, Endogenous networks, social games, and evolution, Games and Economic Behavior 55, 112-130.
- [28] Hojman, D. A., and A. Szeidl, 2008, Core and periphery in networks, Journal of Economic Theory 139, 295-309.
- [29] Holden, C. W., and A. Subrahmanyam, 1992, Long-lived private information and imperfect competition, Journal of Finance 47, 247-270.
- [30] Hong, H., J. D. Kubik, and J. C. Stein, 2004, Social interaction and stock-market participation, Journal of Finance 49, 137-163.

- [31] Hong, H., J. D. Kubik, and J. C. Stein, 2005, Thy neighbor's portfolio: Word-ofmouth effects in the holdings and trades of money managers, Journal of Finance 60, 2801-2824.
- [32] Ivković, Z., and S. Weisbenner, 2007, Information diffusion effects in individual investors' common stock purchases: Covet thy neighbors' investment choices, Review of Financial Studies 20, 1327-1357.
- [33] Jackson, M. O., 2010, Social and Economics Networks, Princeton University Press, New Jersey.
- [34] Jackson, M. O., and A. Wolinsky, 1996, A strategic model of social and economic networks, Journal of Economic Theory 71, 44-74.
- [35] Kyle, A., 1985, Continuous auctions and insider trading, Econometrica 56, 129-176.
- [36] Nanda, V., Z. J. Wang, and L. Zheng, 2004, Family values and the star phenomenon: Strategies of mutual fund families, Review of Financial Studies 17, 667-698.
- [37] Ou-Yang, H., 2003, Optimal contracts in a continuous-time delegated portfolio management problem, Review of Financial Studies 16, 173-208.
- [38] Ozsoylev, H., and J. Walden, 2011, Asset pricing in large information networks, Journal of Economic Theory 146, 2252-2280.
- [39] Palomino, F., and A. Prat, 2003, Risk taking and optimal contracts for money managers, Rand Journal of Economics 34, 113-137.
- [40] Pollet, J. M., and M. Wilson, 2008, How does size affect mutual fund behavior? Journal of Finance 63, 2941-2969.

- [41] Shapley, L. S., 1953, A value for n-person games, Annals of Mathematical Studies 28, 307-317.
- [42] Stein, J. C., 2008, Conversations among competitors, American Economic Review 98, 2150-2162.
- [43] Vayanos, D., and P. Woolley, 2013, An institutional theory of momentum and reversal, Review of Financial Studies 26, 1087-1145.
- [44] Zawadowski, A., 2013, Entangled financial systems, Review of Financial Studies 26, 1291-1323.

Appendix

Proof of Proposition 1: Our proof has two steps. In Step 1, we study how an informed trader chooses her demand, conditional on the price and all the other informed traders' demands being given by Proposition 1. In Step 2, we study how the market maker sets up the price, conditional on all informed investors' demands being given by Proposition 1.

<u>Step 1:</u> Consider an informed trader j. Suppose that the price, P(NA), and the demands of all other informed investors, $D_i(NA)$, $\forall i \neq j$, are given by Proposition 1. Then, her payoff from trading can be expressed as:

$$\Pi_{j}(S_{j}) = E\Big[D_{j}\Big(\bar{F} + X - P(NA)\Big)|S_{j}\Big]$$

$$= E\Big[D_{j}\Big(X - \lambda(NA)(D_{j} + \sum_{i \neq j} D_{i}(NA) + Z)\Big)|S_{j}\Big]$$

$$= E\Big[D_{j}\Big(1 - \lambda(NA)\sum_{i \neq j} \theta_{i}(NA)\Big)X - \lambda(NA)D_{j}^{2}|S_{j}\Big]$$

$$= D_{j}\Big(1 - \lambda(NA)\sum_{i \neq j} \theta_{i}(NA)\Big)E\Big[X|S_{j}\Big] - \lambda(NA)D_{j}^{2}$$

$$= D_{j}\Big(1 - \lambda(NA)\sum_{i \neq j} \theta_{i}(NA)\Big)\frac{S_{j}}{1 + v_{j}} - \lambda(NA)D_{j}^{2}.$$

The first order condition implies

$$D_j(NA) = \frac{1 - \lambda(NA) \sum_{i \neq j} \theta_i(NA)}{2\lambda(NA)} \frac{S_j}{1 + v_j} = \frac{\sigma_z/K(NA)}{1 + 2v_j} S_j,$$

where the last equality follows by substituting the expressions of $\lambda(NA)$ and $\theta_i(NA)$, $\forall i \neq j$ from Proposition 1. The second order condition holds obviously. Therefore, we can write $D_j(NA) = \theta_j(NA)S_j$, where $\theta_j(NA) = \frac{\sigma_z/K(NA)}{1+2v_j}$. Step 2: Consider the market maker. Suppose that the demands of all informed traders, $D_j(NA)$, $\forall j$, are given by Proposition 1. Then, the total demand can be expressed as:

$$D = \sum_{j} D_j(NA) + Z = \sum_{j} \left(\theta_j(NA) S_j \right) + Z = X \sum_{j} \theta_j(NA) + \sum_{j} \left(\theta_j(NA) \epsilon_j \right) + Z.$$

Substituting into the price equation gives

$$P = \bar{F} + E \Big[X | D = X \sum_{j} \theta_{j}(NA) + \sum_{j} \Big(\theta_{j}(NA) \epsilon_{j} \Big) + Z \Big]$$
$$= \bar{F} + \frac{\sum_{j} \theta_{j}(NA)}{\Big(\sum_{j} \theta_{j}(NA)\Big)^{2} + \sum_{j} \Big(\theta_{j}(NA)^{2} v_{j} \Big) + \sigma_{z}^{2}} D.$$

Therefore, we can write $P(NA) = \overline{F} + \lambda(NA)D$, and

$$\begin{split} \lambda(NA) &= \frac{\sum_{j} \theta_{j}(NA)}{(\sum_{j} \theta_{j}(NA))^{2} + \sum_{j} \left(\theta_{j}(NA)^{2}v_{j}\right) + \sigma_{z}^{2}} \\ &= \frac{\sum_{j} \frac{\sigma_{z}/K(NA)}{1 + 2v_{j}}}{\left(\sum_{j} \frac{\sigma_{z}/K(NA)}{1 + 2v_{j}}\right)^{2} + \sum_{j} \left(\left(\frac{\sigma_{z}/K(NA)}{1 + 2v_{j}}\right)^{2}v_{j}\right) + \sigma_{z}^{2}} \\ &= \frac{K(NA)/\sigma_{z}}{1 + \sum_{j} \frac{1}{1 + 2v_{j}}}, \end{split}$$

where the last equality follows by substituting into the expression of K(NA).

Q.E.D.

Proof of Lemma 1: It suffices to show that the distribution of X conditional on S_j , $\forall j \in A$, is identical to the distribution of X conditional on S_A . Note that $X, S_j \ (\forall j \in A)$, and S_A are normally distributed. So the conditional distribution of X must be a normal distribution. We just need to show that

$$E[X|S_j, \forall j \in A] = \frac{S_A}{1 + v_A} = E[X|S_A],$$

$$\operatorname{Var}[X|S_j, \forall j \in A] = \frac{v_A}{1 + v_A} = \operatorname{Var}[X|S_A],$$

which follows immediately after we write down the joint distribution of $X, S_j \ (\forall j \in A)$, and S_A .

Proof of Proposition 2: Consider the coalition as a hypothetical informed trader, indicated by A, who observes a private signal S_A . The remaining proof is identical to the proof of Proposition 1.

Proof of Corollary 1: (i) It follows from Proposition 2 that

$$\begin{aligned} \operatorname{Var}(X|P(A)) &= \operatorname{Var}\Big[X|D = D_A(A) + \sum_{j \notin A} D_j(A) + Z\Big] \\ &= \operatorname{Var}(X) - \lambda(A)\operatorname{Cov}(X, D) \\ &= 1 - \lambda(A)\Big(\theta_A(A) + \sum_{j \notin A} \theta_j(A)\Big) \\ &= 1 - \frac{K(A)/\sigma_z}{1 + \frac{1}{1 + 2v_A} + \sum_{j \notin A} \frac{1}{1 + 2v_j}} \Big(\frac{\sigma_z/K(A)}{1 + 2v_A} + \sum_{j \notin A} \frac{\sigma_z/K(A)}{1 + 2v_j}\Big) \\ &= \frac{1}{1 + \frac{1}{1 + 2v_A} + \sum_{j \notin A} \frac{1}{1 + 2v_j}}.\end{aligned}$$

Similarly,

$$\operatorname{Var}(X|P(A')) = \frac{1}{1 + \frac{1}{1 + 2v'_A} + \sum_{j \notin A'} \frac{1}{1 + 2v_j}}$$

For $\operatorname{Var}(X|P(A')) > \operatorname{Var}(X|P(A))$, it suffices to show that

$$\left(\frac{1}{1+2v_{A'}} + \sum_{j \notin A'} \frac{1}{1+2v_{j}}\right) - \left(\frac{1}{1+2v_{A}} + \sum_{j \notin A} \frac{1}{1+2v_{j}}\right)$$

$$= \frac{1}{1+2v_{A'}} - \left(\frac{1}{1+2v_{A}} + \frac{1}{1+2v_{b}}\right)$$

$$= -\frac{v_{A}(1+2v_{b}) + v_{b}(1+2v_{A})}{(v_{A}+v_{b}+2v_{A}v_{b})(1+2v_{A})(1+2v_{b})}$$

$$< 0, \qquad (3)$$

where the second equality follows from $v_{A'} = 1/(\frac{1}{v_A} + \frac{1}{v_b})$.

(ii) Denote $G \equiv \sum_{j \notin A'} \frac{1 + v_j}{(1 + 2v_j)^2}$ and $Q \equiv \sum_{j \notin A'} \frac{1}{1 + 2v_j}$. It is straightforward to show that G > Q/2.

Note that

$$\begin{split} \lambda(A')^2 - \lambda(A)^2 &= \frac{K(A')^2 / \sigma_z^2}{\left(1 + \frac{1}{1 + 2v_{A'}} + Q\right)^2} - \frac{K(A)^2 / \sigma_z^2}{\left(1 + \frac{1}{1 + 2v_A} + \frac{1}{1 + 2v_b} + Q\right)^2}, \\ &\propto \frac{\frac{1 + v_{A'}}{\left(1 + 2v_{A'}\right)^2} + G}{\left(1 + \frac{1}{1 + 2v_{A'}} + Q\right)^2} - \frac{\frac{1 + v_A}{\left(1 + 2v_A\right)^2} + \frac{1 + v_b}{\left(1 + 2v_b\right)^2} + G}{\left(1 + \frac{1}{1 + 2v_b} + Q\right)^2}, \\ &> \frac{\frac{1 + v_{A'}}{\left(1 + 2v_{A'}\right)^2} + Q/2}{\left(1 + \frac{1}{1 + 2v_{A'}} + Q\right)^2} - \frac{\frac{1 + v_A}{\left(1 + 2v_A\right)^2} + \frac{1 + v_b}{\left(1 + 2v_b\right)^2} + Q/2}{\left(1 + \frac{1}{1 + 2v_b} + Q\right)^2}, \end{split}$$

where the inequality follows from G > Q/2 and Eq. (3).

For $\lambda(A') > \lambda(A)$, it suffices to show that

$$\begin{split} \eta &\equiv & \Big[\frac{1+v_{A'}}{(1+2v_{A'})^2} + Q/2 \Big] (1 + \frac{1}{1+2v_A} + \frac{1}{1+2v_b} + Q)^2 \\ & & - \Big[\frac{1+v_A}{(1+2v_A)^2} + \frac{1+v_b}{(1+2v_b)^2} + Q/2 \Big] (1 + \frac{1}{1+2v_{A'}} + Q)^2 \\ & > & 0. \end{split}$$

After substituting the expression of $v_{A'} = 1/(\frac{1}{v_A} + \frac{1}{v_b})$, write

$$\eta = \eta_1 + 2Q\eta_2 + Q^2\eta_3,$$

where

$$\begin{aligned} \eta_1 &\propto (v_A + v_b + v_A v_b) \Big[4(v_b + 1)v_A^2 + (4v_b^2 + 1)v_A + 4v_b^2 + v_b \Big], \\ \eta_2 &\propto (16v_b^3 + 24v_b^2 + 10v_b + 2)v_A^3 + (24v_b^3 + 20v_b^2 + 4v_b + \frac{1}{4})v_A^2 \\ &+ (10v_b^3 + 4v_b^2 + \frac{1}{2}v_b)v_A + 2v_b^3 + \frac{1}{4}v_b^2, \\ \eta_3 &\propto (32v_b^3 + 28v_b^2 + 8v_b + 1)v_A^3 + (28v_b^3 + 16v_b^2 + 2v_b)v_A^2 + (8v_b^3 + 2v_b^2)v_A + v_b^3 \end{aligned}$$

It is obvious that $\eta_1, \eta_2, \eta_3 > 0$. Therefore, $\eta > 0$ and $\lambda(A') > \lambda(A)$.

(iii) Note that $\forall \alpha, \beta > 0$. We have the following inequality:

$$f(\alpha, \beta) \equiv \beta(\alpha + \beta + \alpha\beta)(1 + 2\alpha)^2 - (1 + \alpha)(\alpha + \beta + 2\alpha\beta)^2$$
$$= -\alpha^3(4 + 3\beta) - \alpha^2(1 + 5\beta) - \alpha\beta$$
$$< 0.$$

Substituting $v_{A'} = 1 / (\frac{1}{v_A} + \frac{1}{v_b})$ yields

$$K(A')^{2} - K(A)^{2} = \frac{1 + v_{A'}}{(1 + 2v_{A'})^{2}} - \frac{1 + v_{A}}{(1 + 2v_{A})^{2}} - \frac{1 + v_{b}}{(1 + 2v_{b})^{2}}$$

$$\propto f(v_{A}, v_{b})(1 + 2v_{A})^{2} + f(v_{b}, v_{A})(1 + 2v_{b})^{2}$$

$$< 0.$$

It follows from K(A') < K(A) that $\forall j \notin A'$,

$$\theta_j(A') = \frac{\sigma_z/K(A')}{1+2v_j} > \frac{\sigma_z/K(A)}{1+2v_j} = \theta_j(A).$$

It follows from K(A') < K(A) and $v_{A'} < v_A$ that

$$\theta_{A'}(A') = \frac{\sigma_z/K(A')}{1+2v_{A'}} > \frac{\sigma_z/K(A)}{1+2v_A} = \theta_A(A).$$
 Q.E.D.

Proof of Corollary 2: (i) It follows from $\theta_j(A) > \theta_j(NA)$ and $\lambda(A) > \lambda(NA)$ (see Corollary 1) that

$$\Gamma_j(A) = E\Pi_j(A) - E\Pi_j(NA) = \lambda(A)\theta_j^2(A)(1+v_j) - \lambda(NA)\theta_j^2(NA)(1+v_j) > 0.$$

(ii) It follows from $\lambda(A) > \lambda(NA)$ (see Corollary 1) that

$$\Gamma_{\rm L}(A) = E\Pi_{\rm L}(A) - E\Pi_{\rm L}(NA) = (\lambda(NA) - \lambda(A)) \cdot \sigma_z^2 < 0.$$

Q.E.D.

Proof of Corollary 3: (i) Denote $v_{23} = 1/(\frac{1}{v_2} + \frac{1}{v_3})$. Write

$$\begin{split} &\Gamma_A(A = \{2,3\}) \\ = & E\Pi_A(A = \{2,3\}) - \left[E\Pi_2(NA) + E\Pi_3(NA)\right] \\ = & \frac{\sigma_z}{\sqrt{\frac{1+v_{23}}{(1+2v_{23})^2} + \frac{1+v_1}{(1+2v_1)^2}}} \frac{\frac{1+v_{23}}{(1+2v_{23})^2}}{1 + \frac{1}{1+2v_{23}} + \frac{1}{1+2v_1}} \\ & - \frac{\sigma_z}{\sqrt{\frac{1+v_1}{(1+2v_1)^2} + \frac{1+v_2}{(1+2v_2)^2} + \frac{1+v_3}{(1+2v_3)^2}}} \frac{\frac{1+v_2}{(1+2v_2)^2} + \frac{1+v_3}{(1+2v_3)^2}}{1 + \frac{1}{1+2v_1} + \frac{1}{1+2v_2} + \frac{1}{1+2v_3}}. \end{split}$$

Denote $v_2 = v_1 + a$ and $v_3 = v_1 + b$ where $b \ge a \ge 0$. We can show that $\Gamma_A(A = \{2, 3\})$ is proportional to a polynomial of v_1 , a, and b, which is non-positive. (We derive this polynomial using Mathematica. It is six pages long and available upon request from the authors.) Therefore, $\Gamma_A(A = \{2, 3\}) \le 0$.

authors.) Therefore, $\Gamma_A(A = \{2, 3\}) \le 0$. (ii) Denote $v_{12} = 1/(\frac{1}{v_1} + \frac{1}{v_2})$, and $v_{13} = 1/(\frac{1}{v_1} + \frac{1}{v_3})$. Write

$$\begin{split} &\Gamma_A(A=\{1,2\})\\ = & E\Pi_A(A=\{1,2\}) - \left[E\Pi_1(NA) + E\Pi_2(NA)\right]\\ = & \frac{\sigma_z}{\sqrt{\frac{1+v_{12}}{(1+2v_{12})^2} + \frac{1+v_3}{(1+2v_3)^2}}} \frac{\frac{1+v_{12}}{(1+2v_{12})^2}}{1+\frac{1}{1+2v_{12}} + \frac{1}{1+2v_3}}\\ & - \frac{\sigma_z}{\sqrt{\frac{1+v_1}{(1+2v_1)^2} + \frac{1+v_2}{(1+2v_2)^2}} + \frac{1+v_3}{(1+2v_3)^2}} \frac{\frac{1+v_1}{(1+2v_1)^2} + \frac{1+v_2}{(1+2v_2)^2}}{1+\frac{1}{1+2v_1} + \frac{1}{1+2v_2} + \frac{1}{1+2v_3}}, \end{split}$$

and

$$\begin{split} &\Gamma_A(A = \{1,3\}) \\ = & E\Pi_A(A = \{1,3\}) - \left[E\Pi_1(NA) + E\Pi_3(NA)\right] \\ = & \frac{\sigma_z}{\sqrt{\frac{1+v_{13}}{(1+2v_{13})^2} + \frac{1+v_2}{(1+2v_2)^2}}} \frac{\frac{1+v_{13}}{(1+2v_{13})^2}}{1 + \frac{1}{1+2v_{13}} + \frac{1}{1+2v_2}} \\ & - \frac{\sigma_z}{\sqrt{\frac{1+v_1}{(1+2v_1)^2} + \frac{1+v_2}{(1+2v_2)^2}}} \frac{\frac{1+v_1}{(1+2v_1)^2} + \frac{1+v_3}{(1+2v_3)^2}}{1 + \frac{1}{1+2v_1} + \frac{1}{1+2v_2} + \frac{1}{1+2v_3}}. \end{split}$$

We can show using Mathematica Symbolic Computing that $\Gamma_A(A = \{1, 2\}) \ge \Gamma_A(A = \{1, 3\}).$

(iii) Similarly to the proof for Part (ii), we can also show using Mathematica Symbolic Computing that

$$\Gamma_1(A = \{1, 2\}) \equiv \frac{1/v_1}{1/v_1 + 1/v_2} \Gamma_A(A = \{1, 2\})$$

$$\geq \Gamma_1(A = \{1, 3\}) \equiv \frac{1/v_1}{1/v_1 + 1/v_3} \Gamma_A(A = \{1, 3\}).$$

Q.E.D.



Figure 1: A Partial Coalition When J = 3

There are J = 3 informed traders. Their information qualities satisfy $1 = v_1 \le v_2 \le v_3 \le 40$. In the shaded area,

 $\Gamma_A(A = \{1, 2\}) \ge \max(0, \Gamma_A(A = \{1, 3\}), \Gamma_A(A = \{2, 3\})),$

so a partial coalition, $A = \{1, 2\}$, is admissible.



Figure 2: A Partial Coalition When J = 3: Stability

There are J = 3 informed traders. Their information qualities satisfy $1 = v_1 \le v_2 \le v_3 \le 40$. The shaded area satisfies

$$\Gamma_A(A = \{1, 2\}) \ge \max(0, \Gamma_A(A = \{1, 3\}), \Gamma_A(A = \{2, 3\})).$$

The dark-shaded area further satisfies

$$\Gamma_2(A = \{1, 2\}) = \frac{1/v_2}{1/v_1 + 1/v_2} \Gamma_A(A = \{1, 2\}) \ge \Gamma_2(A = \{1, 3\}).$$

A partial coalition, $A = \{1, 2\}$, is stable in the dark-shaded area, but fragile in the light-shaded area.



Figure 3: A Partial Coalition When J = 10: Case 1

There are J = 10 informed traders. Their information qualities satisfy $1 = v_1 \le v_2 \le v_3 \le 20$ and $v_j = 50, \forall j > 3$. In the shaded area,

$$\Gamma_A(A = \{1, 2\}) \ge \max(0, \Gamma_A(A = \{1, 3\}), \Gamma_A(A = \{2, 3\})),$$

so a partial coalition, $A = \{1, 2\}$, is admissible.



Figure 4: A Partial Coalition When J = 10: Case 2

There are J = 10 informed traders. Their information qualities satisfy $1 = v_1 \le v_2 \le v_3 \le 20$ and $v_j = 35$, $\forall j > 3$. In the shaded area,

$$\Gamma_A(A = \{1, 2\}) \ge \max(0, \Gamma_A(A = \{1, 3\}), \Gamma_A(A = \{2, 3\})),$$

so a partial coalition, $A = \{1, 2\}$, is admissible.



(a): The synergy of Hypothetical Partial Coalition $A = \{1, 2\}, \Gamma_A(A = \{1, 2\})$



Figure 5: Welfare Effects of Hypothetical Partial Coalition

There are J = 10 informed traders, each of whom has the same-quality information, $v_j = V, \forall j$. Let $\sigma_z^2 = 100$. Panel (a) plots the synergy for a hypothetical partial coalition $A = \{1, 2\}, \Gamma_A(A = \{1, 2\})$, depending on V. Panel (b) plots the profit change of an independent informed trader due to the coalition, $\Gamma_j(A = \{1, 2\}), \forall j \ge 3$, depending on V.



Figure 6: Minimum Coalition Size: Case 1

There are J = 40 informed traders, each of whom has the same-quality information, $v_j = V, \forall j$. We assume that there can be 1, 2, or 3 coalitions. Because of symmetry, each coalition has the same number of informed players. The minimum size of an admissible coalition, \underline{m} , is given by:

$$\min_{m} \quad m \\ s.t. \quad \Gamma_{A}(A = \{1, 2, ..., m\}) \ge 0.$$

This figure plots the minimum coalition size, \underline{m} , depending on V.



Figure 7: Minimum Coalition Size: Case 2

There can be J = 20, 30, or 40 informed traders, each of whom has the same-quality information, $v_j = 2.5, \forall j$. We assume that there can be several coalitions. Because of symmetry, each coalition has the same number of informed players. The minimum size of an admissible coalition, \underline{m} , is given by:

$$\min_{m} \quad m \\ s.t. \quad \Gamma_A(A = \{1, 2, ..., m\}) \ge 0.$$

This figure plots the minimum coalition size, \underline{m} , depending on the number of coalitions.



(b) Optimal Number of Coalitions



There can be J = 30 or 40 informed traders, each of whom has the same-quality information, $v_j = V$, $\forall j$. By symmetry, there can be *n* coalitions, each of which has *m* informed traders. The optimal $\{m, n\}$ are given by:

$$\max_{m,n} \quad \frac{1}{m} \left[\Gamma_A(A = \{1, 2, ..., m\}) - C(A(m)) \right],$$

where $C(A(m)) = 10^{-4}(m-1)^2$. This figure plots the optimal $\{m, n\}$ depending on V.



(b) Optimal Number of Coalitions

Figure 9: Optimal Coalition Structure with Two Isolated Groups of Informed Traders: Case 1

There are two isolated groups. Each group has 40 informed traders with the same-quality information, represented by V_1 and V_2 . Let $V_1 = 1$ and $V_2 > 1$. By symmetry, in group 1 (2), there can be n_1 (n_2) coalitions, each of which has m_1 (m_2) informed traders. The optimal $\{m_1, m_2, n_1, n_2\}$ are given by:

$$\max_{m_1, m_2, n_1, n_2} \frac{1}{m_1} \Big[\Gamma_A(A = \{1, ..., m_1\}) - C(A(m_1)) \Big] + \frac{1}{m_2} \Big[\Gamma_A(A = \{1, ..., m_2\}) - C(A(m_2)) \Big],$$

where $C(A(m)) = 10^{-4}(m-1)^2$. This figure plots the optimal $\{m_1, m_2, n_1, n_2\}$ depending on V_2 .



Figure 10: Optimal Coalition Structure with Two Isolated Groups of Informed Traders: Case 2

There are two isolated groups. Group 1 (2) has $J_1(J_2)$ informed traders. Let $J_1 > 100$ and $J_2 = 100$. Informed traders in both groups have the same-quality information, $V_1 = V_2 = 5$. By symmetry, in group 1 (2), there can be $n_1(n_2)$ coalitions, each of which has $m_1(m_2)$ informed traders. The optimal $\{m_1, m_2, n_1, n_2\}$ are given by:

$$\max_{m_1, m_2, n_1, n_2} \frac{1}{m_1} \Big[\Gamma_A(A = \{1, ..., m_1\}) - C(A(m_1)) \Big] + \frac{1}{m_2} \Big[\Gamma_A(A = \{1, ..., m_2\}) - C(A(m_2)) \Big],$$

where $C(A(m)) = 10^{-5}(m-1)^2$. This figure plots the optimal $\{m_1, m_2, n_1, n_2\}$ depending on J_1 .