Online Appendix

Managing Weather Risk with a Neural Network-Based Index Insurance *

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Management Science, accepted

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A An example of an overfitted solution to problem (3)

Let's consider a special case of problem (3) as an illustrating example. Let $\{(\boldsymbol{x}_j, y_j)\}_{j=1,\dots,n}$ be a realized sample of $(\boldsymbol{X}, Y)$. Consider the minimization problem:

$$\begin{cases} \min_{I \in \mathcal{I}} & -\dfrac{1}{n}\displaystyle\sum_{j=1}^{n} U[w - y_j + I(\boldsymbol{x}_j) - \pi_e(I)] \\ \text{s.t.} & P_L \le \pi_e(I) = \dfrac{\lambda}{n}\displaystyle\sum_{j=1}^{n} I(\boldsymbol{x}_j) \le P_U, \end{cases} \tag{A.1}$$

where $\mathcal{I} := \{I : \mathbb{R}^p \mapsto \mathbb{R}^+ | I \text{ is measurable}\}$. For simplicity we also replace the budget constraint by:

$$P_L = P_U = P = \lambda \frac{1}{n}\sum_{j=1}^{n} I(\boldsymbol{x}_j).$$

Then we have the following proposition:

Proposition 1 (Jensen's inequality)**.** *For any concave utility function U and any deterministic function I such that $P = \lambda \frac{1}{n}\sum_{j=1}^{n} I(\boldsymbol{x}_j)$, we have:*

$$\begin{aligned} \frac{1}{n}\sum_{j=1}^{n} U[w - y_j + I(\boldsymbol{x}_j) - P] &\le U\left[\frac{1}{n}\sum_{j=1}^{n}\{w - y_j + I(\boldsymbol{x}_j) - P\}\right] \\ &= U\left[w - \frac{1}{n}\sum_{j=1}^{n} y_j + \frac{P}{\lambda} - P\right], \end{aligned}$$

with equality if and only if $I(\boldsymbol{x}_j) - y_j$ is a constant. Therefore the optimal solution I^* is given by,

$$I^*(\boldsymbol{x}) = \begin{cases} y_j + P/\lambda - \frac{1}{n}\sum\limits_{j=1}^{n} y_j, & \textit{if } \boldsymbol{x} = \boldsymbol{x}_j, j = 1, 2, ..., n, \\ \textit{any arbitrary number}, & \textit{otherwise}. \end{cases} \tag{A.2}$$

This solution is "overfitted": although it mathematically optimizes problem (A.1), it says nothing about what the amount of indemnity should be for a new data sample. In fact, the problem comes from the fact that the admissible functional space $\mathcal{I}$ is too large and contains functions that are not constrained, or smooth enough. On the other hand, for instance, we constrain the space $\mathcal{I}$ to be the space of linear functions, then equations (A.2) cannot be satisfied for all indices $j = 1, ..., n$. Such a solution is too smooth, and usually result in a poor fit. Thus the challenge is to find a trade-off between these two extreme cases, that is, to propose suitable functional constraints on the $\mathcal{I}$.

B Feasible sets

We want to choose a feasible set, $\mathcal{I}_0$, which balances between flexibility and stability. $\mathcal{I}_0$ should be large enough to include candidate payoff functions that capture intricate (nonlinear, nonmonotonic) relationships between the high-dimensional indices and losses, yet $\mathcal{I}_0$ should also exclude "ill-behaved" ones in $\mathcal{I}$, which are sensitive to the sample data and cannot be an appropriate insurance contract. Such trade-off is illustrated in Figure B.1. The blue star in $\mathcal{I}$ is the global optimal contract, which may not be obtained.[1] The red triangle illustrates a highly unstable, overfitted contract, which we want to avoid. The dotted-grey circle area, $\tilde{\mathcal{I}}_0 \subset \mathcal{I}$, is a set of piecewise linear contracts. Although quite stable, $\tilde{\mathcal{I}}_0$ is far away from the blue star due to its restrictive functional form. Our goal is to expand the boundary of the feasible set towards $\mathcal{I}_0$, and obtain the optimal contract that falls within the intersection area, which is represented by the blue diamond. This optimal contract sacrifices a little stability but achieves much more flexibility and hence a large amount of basis risk reduction.

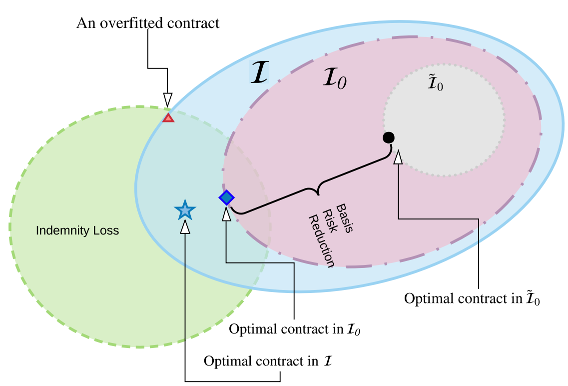
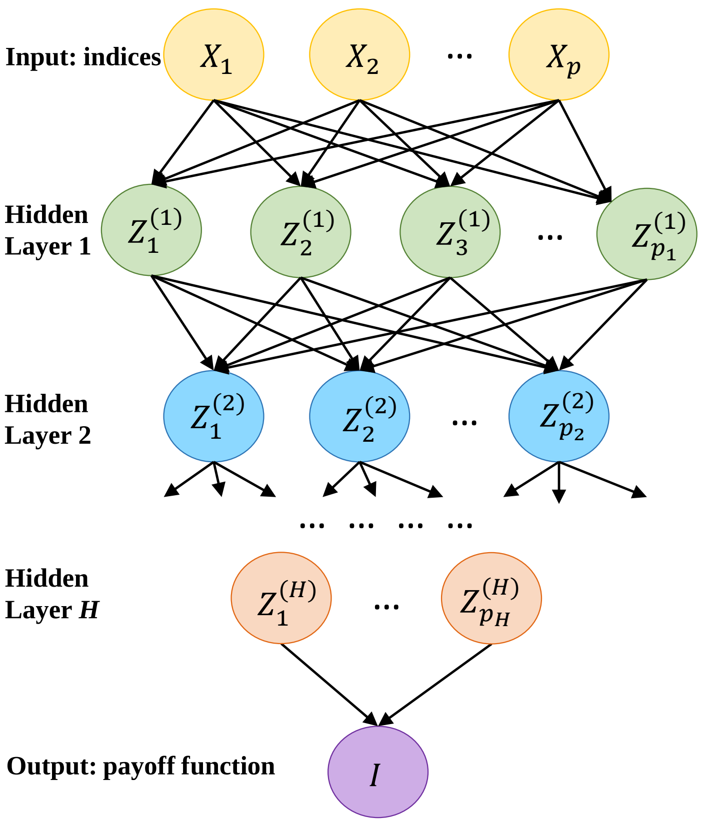


Figure B.1: **Feasible sets and optimal contracts.** This figure compares three different feasible sets and their corresponding optimal contracts. The dashed-green circle area represents the indemnity loss, which is the actual loss experienced by the policyholder. The general feasible set, $\mathcal{I}$, is represented by the solid-blue circle area and the blue star denotes the global optimal contract. The dotted-grey circle area, $\tilde{\mathcal{I}}_0$, is a feasible set of all piecewise linear contracts. The black dot at the edge of $\tilde{\mathcal{I}}_0$ is the optimal piecewise linear contract, i.e., the contract with the smallest basis risk within $\tilde{\mathcal{I}}_0$. The dotted-blue area, $\mathcal{I}_0$, represents the feasible set we explore. Its optimal contract is denoted by the blue diamond. The red triangle illustrates an overfitted contract.

[1] This is due to the fact that we replace the expectation in problem (1) with its empirical counterpart.

C Neural network structure

Figure C.1 illustrates a neural network with H-hidden layers.



$$\boldsymbol{X}_{Input} = \boldsymbol{X}^T = (X_1, X_2, \ldots, X_p)^T$$

$$\boldsymbol{Z}^{(1)} = f_0(\boldsymbol{\alpha}^{(0)} + \boldsymbol{\omega}^{(0)}\boldsymbol{X}_{Input})$$

$$\boldsymbol{Z}^{(2)} = f_1(\boldsymbol{\alpha}^{(1)} + \boldsymbol{\omega}^{(1)}\boldsymbol{Z}^{(1)})$$

$$\boldsymbol{Z}^{(H)} = f_{H-1}(\boldsymbol{\alpha}^{(H-1)} + \boldsymbol{\omega}^{(H-1)}\boldsymbol{Z}^{(H-1)})$$

$$I = f_H(\boldsymbol{\alpha}^{(H)} + \boldsymbol{\omega}^{(H)}\boldsymbol{Z}^{(H)})$$

Figure C.1: **An illustration of neural networks with H-hidden layers.** This is an example of the fully-connected architecture in which neurons between two adjacent layers are fully pairwise connected, but neurons within a layer have no connections. f_h is an activation function; $\boldsymbol{\alpha}^{(h)}$ and $\boldsymbol{\omega}^{(h)}$ are parameters of the linear combination, $h = 1, 2, \ldots, H$.

D Algorithm: Solve for the optimal index insurance policy

Output: An optimal index insurance policy I
Input : Index data $\boldsymbol{X}$ and loss data Y

Build and initialize a neural network;
Initialization: $k = 0, \phi_0 = \epsilon_1$, obtain I by solving an unconstrained problem Φ_0;
while $|I - I_{last}| > \epsilon_3$ **or** $g(I) > \epsilon_2$ **or** $|\pi_e(I) - \pi_e(I_{last})| > \epsilon_4$ **do**
  Set $I_{last} = I$, $\pi_e(I_{last}) = \pi_e(I)$;
  Update $k \leftarrow k + 1$;
  Train the neural network and obtain the optimal I for problem $\Phi_k(I)$:

$$\Phi_k(I) = -\frac{1}{n}\sum_{j=1}^{n} U\big(w - y_j + I(\boldsymbol{x}_j) - \pi_e(I)\big) + \phi_k \cdot g(I),$$

  where the loss function is customized according to $\Phi_k(I)$ and I_{last} is set to the initial value of optimization;
  Update $g(I)$ and $\pi_e(I)$;
end
return (I)

Algorithm 1: Solve for the optimal index insurance policy.

E Data Summary

Tables E.1 and E.2 show summary statistics of the 72 weather indices used for empirical analysis. Statistics including mean, standard deviation, minimum, 25^{th} and 75^{th} percentiles, and maximum, are presented. The sample period is 1925-2018.

Table E.1: Summary statistics of weather indices. See descriptions for index variables abbreviations in Table 1.

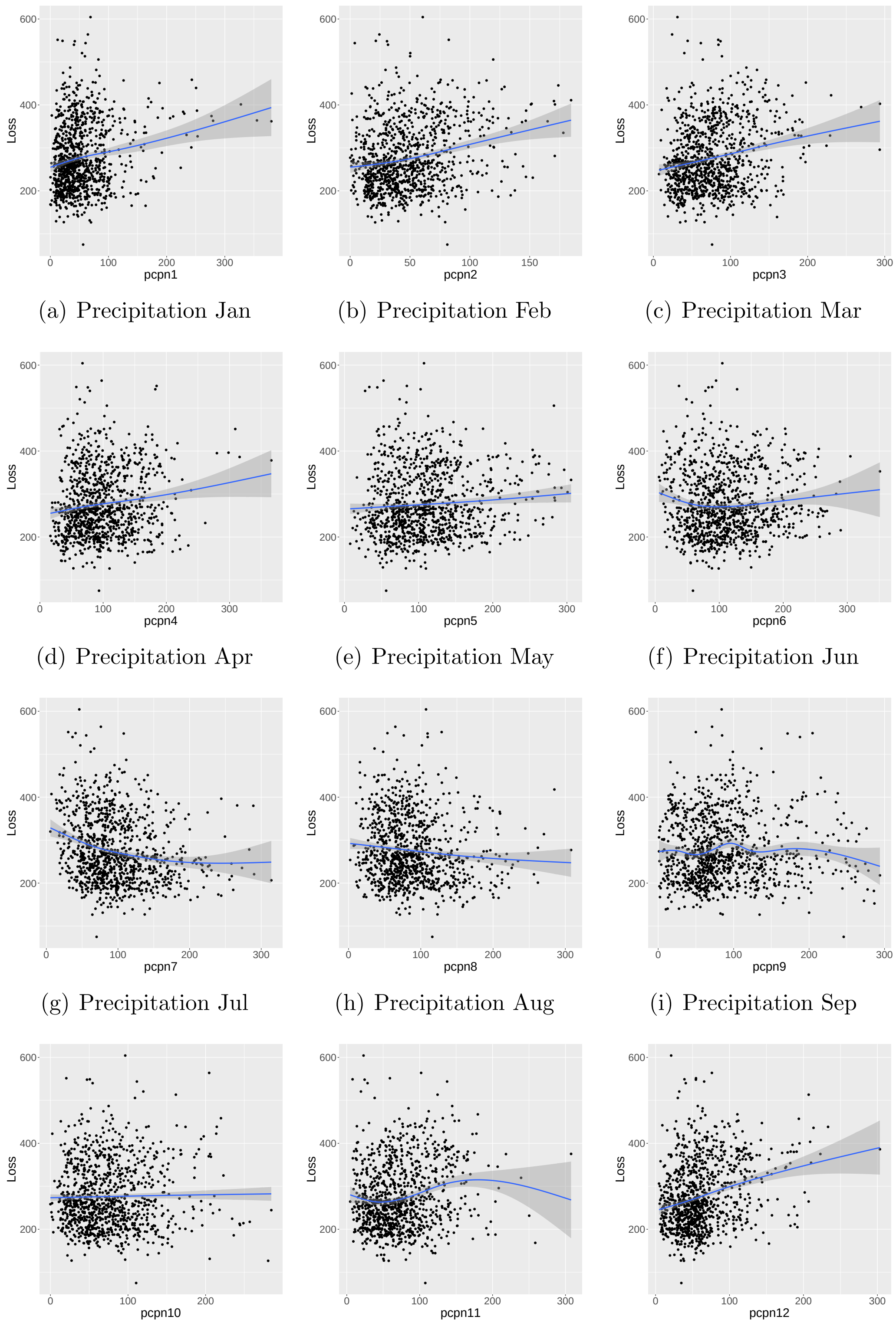
	pcpn1	pcpn2	pcpn3	pcpn4	pcpn5	pcpn6	pcpn7	pcpn8	pcpn9	pcpn10	pcpn11	pcpn12
Mean	53.36	49.69	77.37	98.32	106.95	108.91	94.97	87.04	86.25	74.84	74.97	62.08
StD	41.73	31.35	42.99	47.51	54.25	52.62	47.61	45.58	55.35	45.60	44.94	39.53
Min	0.03	0.06	6.86	16.60	7.71	4.28	3.97	2.30	0.21	0.02	4.28	3.90
Q_{25}	26.11	27.27	45.38	63.49	67.21	71.22	61.45	55.14	47.90	42.48	42.02	35.36
Q_{75}	68.76	65.37	99.57	125.26	133.62	138.28	120.13	109.56	111.79	98.32	100.17	78.26
Max	411.75	260.57	372.58	413.99	365.64	419.15	347.60	371.80	339.07	310.42	322.15	338.16
	dpt1	dpt2	dpt3	dpt4	dpt5	dpt6	dpt7	dpt8	dpt9	dpt10	dpt11	dpt12
Mean	-7.66	-5.91	-1.57	3.64	9.58	14.97	17.61	17.02	12.60	6.00	0.06	-5.13
StD	3.50	3.36	2.72	2.31	2.43	1.81	1.47	1.51	1.87	2.20	2.28	3.18
Min	-20.75	-17.26	-10.10	-3.96	2.64	9.19	12.93	12.49	7.32	-3.59	-8.08	-16.12
Q_{25}	-9.89	-8.19	-3.52	2.03	7.89	13.76	16.55	16.01	11.33	4.70	-1.42	-7.16
Q_{75}	-5.26	-3.46	0.20	5.27	11.26	16.25	18.62	18.08	13.90	7.39	1.69	-2.89
Max	2.11	2.14	7.22	10.09	16.84	20.35	22.35	21.52	18.51	12.57	6.71	4.80
	tmax1	tmax2	tmax3	tmax4	tmax5	tmax6	tmax7	tmax8	tmax9	tmax10	tmax11	tmax12
Mean	1.73	4.24	10.37	17.75	23.59	28.58	30.71	29.74	26.27	19.68	11.11	3.88
StD	3.77	3.87	3.60	2.60	2.36	2.02	2.00	1.99	2.17	2.54	2.86	3.39
Min	-9.45	-8.17	-0.42	9.11	16.72	23.08	24.23	24.04	18.94	9.35	2.53	-7.20
Q_{25}	-0.82	1.58	7.82	15.93	21.90	27.17	29.40	28.39	24.68	18.07	9.16	1.67
Q_{75}	4.42	7.02	12.84	19.62	25.25	29.85	31.89	30.99	27.79	21.19	13.13	6.27
Max	12.51	15.14	21.65	25.03	30.58	35.78	38.13	37.82	33.48	28.53	18.97	13.41

Table E.2: Summary statistics of weather indices (cont'd). See descriptions for index variables abbreviations in Table 1.

	tmin1	tmin2	tmin3	tmin4	tmin5	tmin6	tmin7	tmin8	tmin9	tmin10	tmin11	tmin12
Mean	-7.84	-5.88	-0.77	5.22	10.89	16.11	18.22	17.10	12.86	6.56	0.45	-5.23
StD	3.81	3.71	2.92	2.22	2.17	1.74	1.56	1.75	1.89	2.02	2.30	3.40
Min	-21.01	-19.34	-12.21	-2.61	4.93	10.41	13.32	11.29	7.58	-0.48	-7.93	-17.39
Q_{25}	-10.30	-8.28	-2.73	3.68	9.33	14.94	17.15	15.95	11.53	5.27	-1.01	-7.32
Q_{75}	-5.13	-3.16	1.14	6.80	12.38	17.32	19.30	18.27	14.15	7.92	2.03	-2.89
Max	1.70	3.34	9.21	11.92	18.07	22.11	23.33	24.05	19.71	13.04	8.15	4.00
	vpdmax1	vpdmax2	vpdmax3	vpdmax4	vpdmax5	vpdmax6	vpdmax7	vpdmax8	vpdmax9	vpdmax10	vpdmax11	vpdmax12
Mean	2.86	3.84	6.78	11.81	16.26	21.03	22.87	21.18	18.68	12.94	6.45	3.27
StD	1.09	1.42	2.11	2.43	3.15	4.11	4.90	4.46	3.95	3.05	1.82	1.08
Min	0.81	1.21	2.16	6.19	8.64	13.19	12.41	12.41	9.04	4.49	2.16	0.89
Q_{25}	2.00	2.76	5.28	9.96	14.07	18.07	19.73	18.04	15.92	11.04	5.13	2.44
Q_{75}	3.52	4.71	8.07	13.48	18.17	23.17	25.11	23.57	20.97	14.43	7.58	3.96
Max	7.18	9.80	15.98	19.94	28.31	37.81	44.68	43.10	36.38	28.28	13.71	7.05
	vpdmin1	vpdmin2	vpdmin3	vpdmin4	vpdmin5	vpdmin6	vpdmin7	vpdmin8	vpdmin9	vpdmin10	vpdmin11	vpdmin12
Mean	0.45	0.52	0.78	1.33	1.59	1.87	1.61	1.16	1.10	0.96	0.72	0.48
StD	0.19	0.21	0.29	0.37	0.49	0.72	0.75	0.65	0.51	0.35	0.23	0.19
Min	0.01	0.01	0.05	0.28	0.06	0.08	0.00	0.00	0.00	0.00	0.07	0.01
Q_{25}	0.30	0.35	0.57	1.09	1.28	1.36	1.10	0.70	0.76	0.74	0.55	0.33
Q_{75}	0.60	0.68	0.97	1.56	1.84	2.25	2.01	1.50	1.35	1.16	0.88	0.63
Max	1.05	1.18	2.04	2.98	4.29	6.39	6.13	5.40	3.82	3.18	1.78	1.32

F Nonlinear relationships between production losses and weather indices

This appendix collects scatterplots of all 72 weather indices against the crop losses, using 1,000 random draws from the sample. The blue curve is fitted by a generalized additive model. The shadow area indicates 95% confidence interval. We can see that most weather indices have intricate nonlinear relationships with crop losses, and this complexity could not be adequately captured by linear models that are used by most existing index insurance design framework. The nonlinearity suggests inadequacy of those index insurance with simple structures, and calls for more sophisticated models.



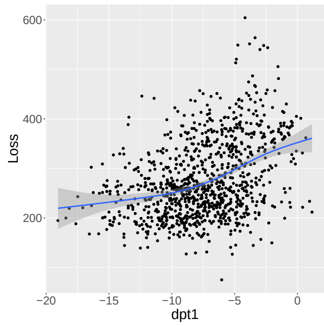
(a) Precipitation Jan (b) Precipitation Feb (c) Precipitation Mar

(d) Precipitation Apr (e) Precipitation May (f) Precipitation Jun

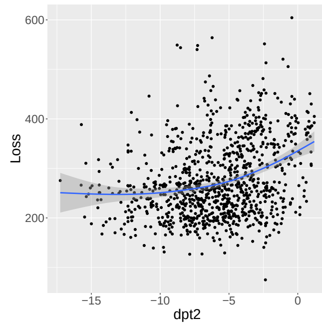
(g) Precipitation Jul (h) Precipitation Aug (i) Precipitation Sep

(j) Precipitation Oct (k) Precipitation Nov (l) Precipitation Dec

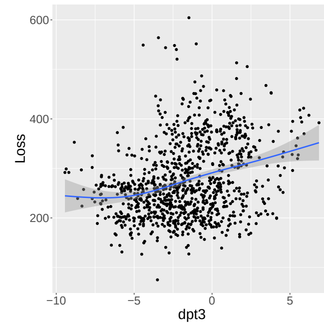
Figure F.1: Scatterplots of precipitation (Jan-Dec) with crop losses.



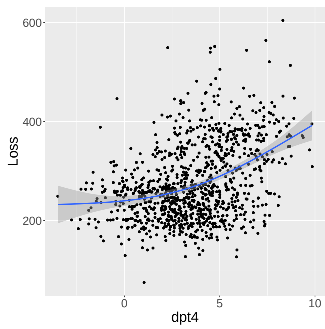
(a) Dew Points Jan



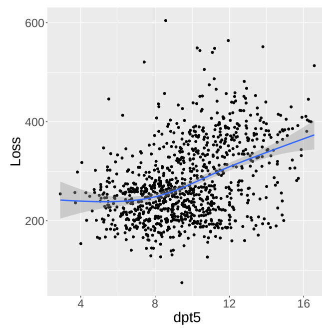
(b) Dew Points Feb



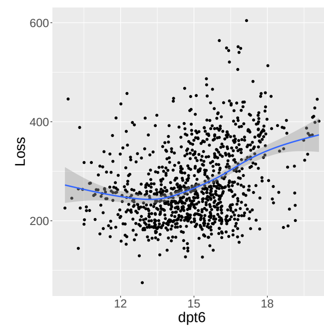
(c) Dew Point Mar



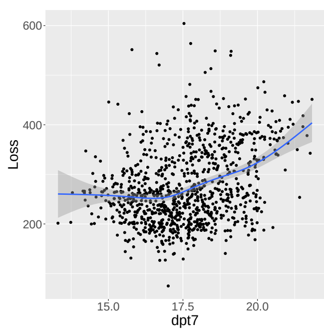
(d) Dew Points Apr



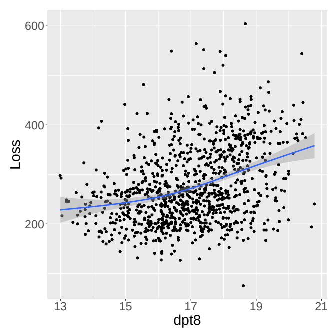
(e) Dew Points May



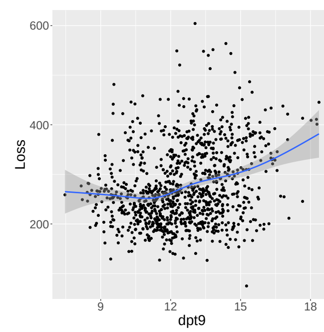
(f) Dew Points Jun



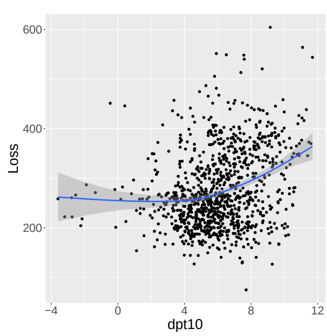
(g) Dew Points Jul



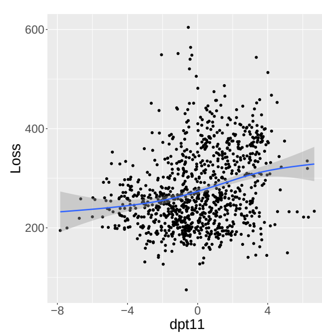
(h) Dew Points Aug



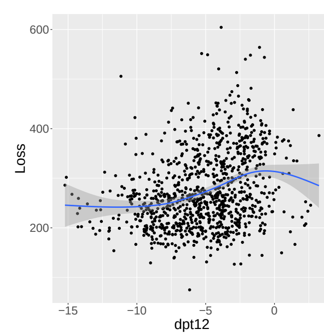
(i) Dew Points Sep



(j) Dew Points Oct

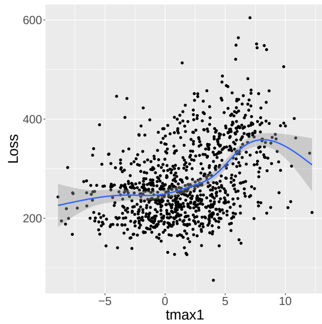


(k) Dew Points Nov

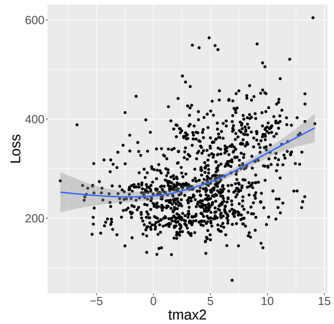


(l) Dew Points Dec

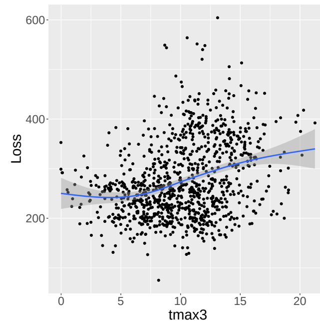
Figure F.2: Scatterplots of dew point temperature (Jan-Dec) with crop losses.



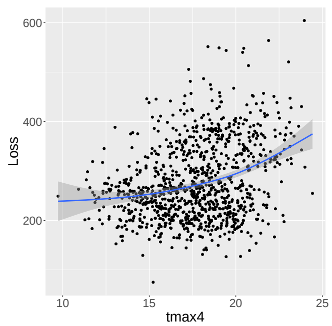
(a) Max Temperature Jan



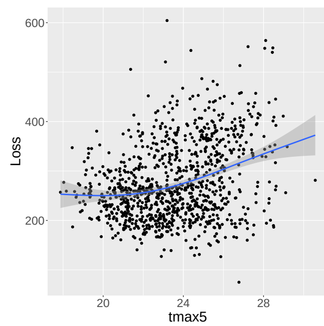
(b) Max Temperature Feb



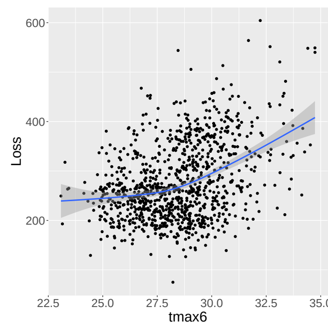
(c) Max Temperature Mar



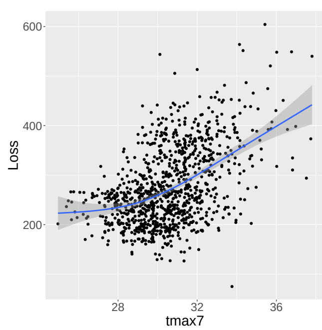
(d) Max Temperature Apr



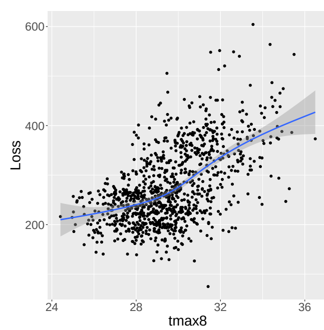
(e) Max Temperature May



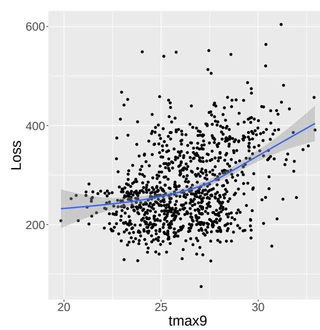
(f) Max Temperature Jun



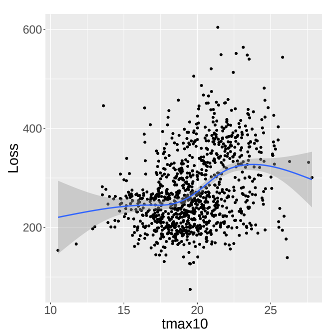
(g) Max Temperature Jul



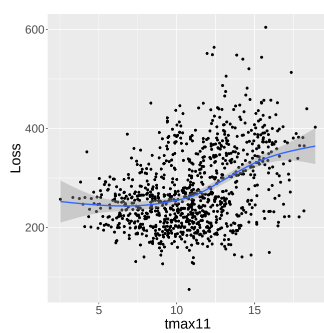
(h) Max Temperature Aug



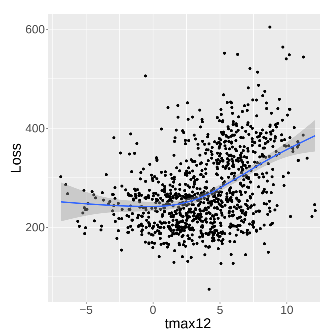
(i) Max Temperature Sep



(j) Max Temperature Oct

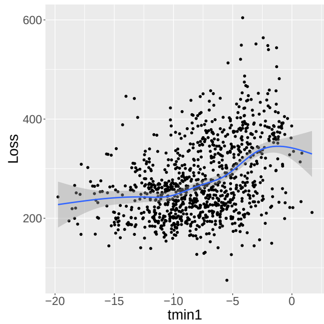


(k) Max Temperature Nov

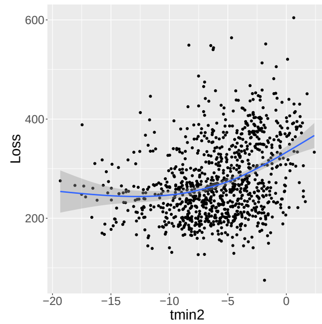


(l) Max Temperature Dec

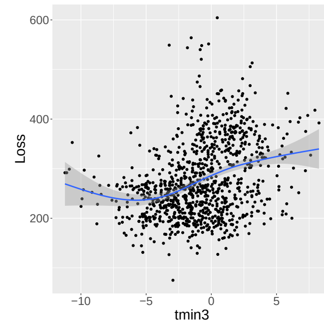
Figure F.3: Scatterplots of maximum temperature (Jan-Dec) with crop losses.



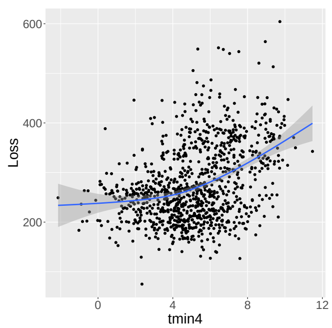
(a) Min Temperature Jan



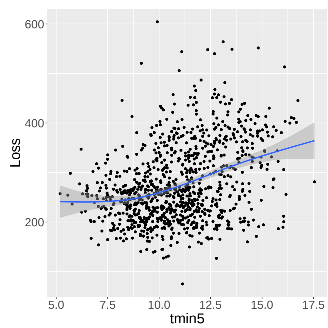
(b) Min Temperature Feb



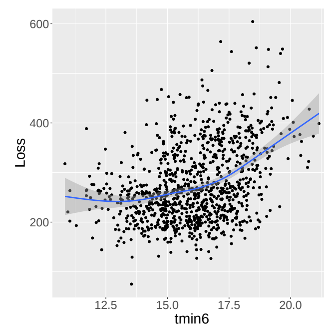
(c) Min Temperature Mar



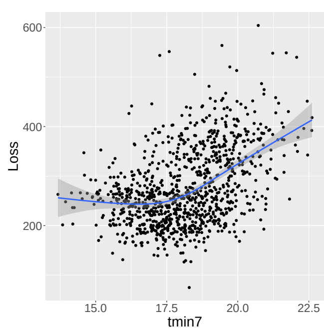
(d) Min Temperature Apr



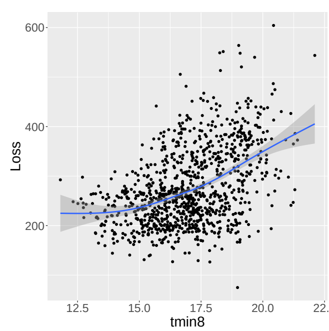
(e) Min Temperature May



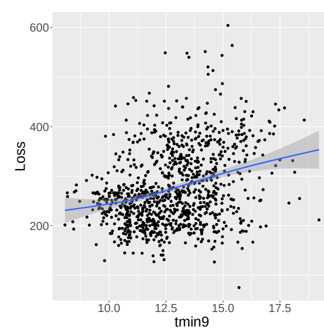
(f) Min Temperature Jun



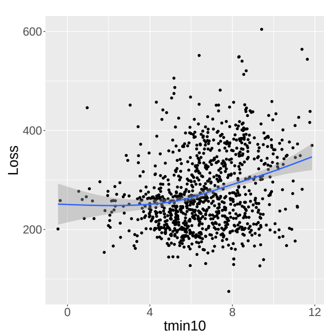
(g) Min Temperature Jul



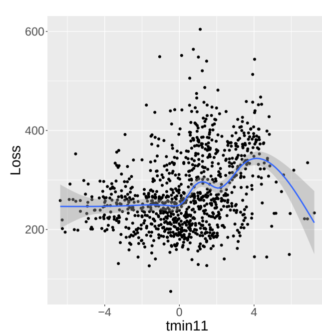
(h) Min Temperature Aug



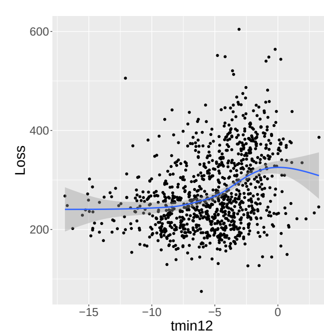
(i) Min Temperature Sep



(j) Min Temperature Oct

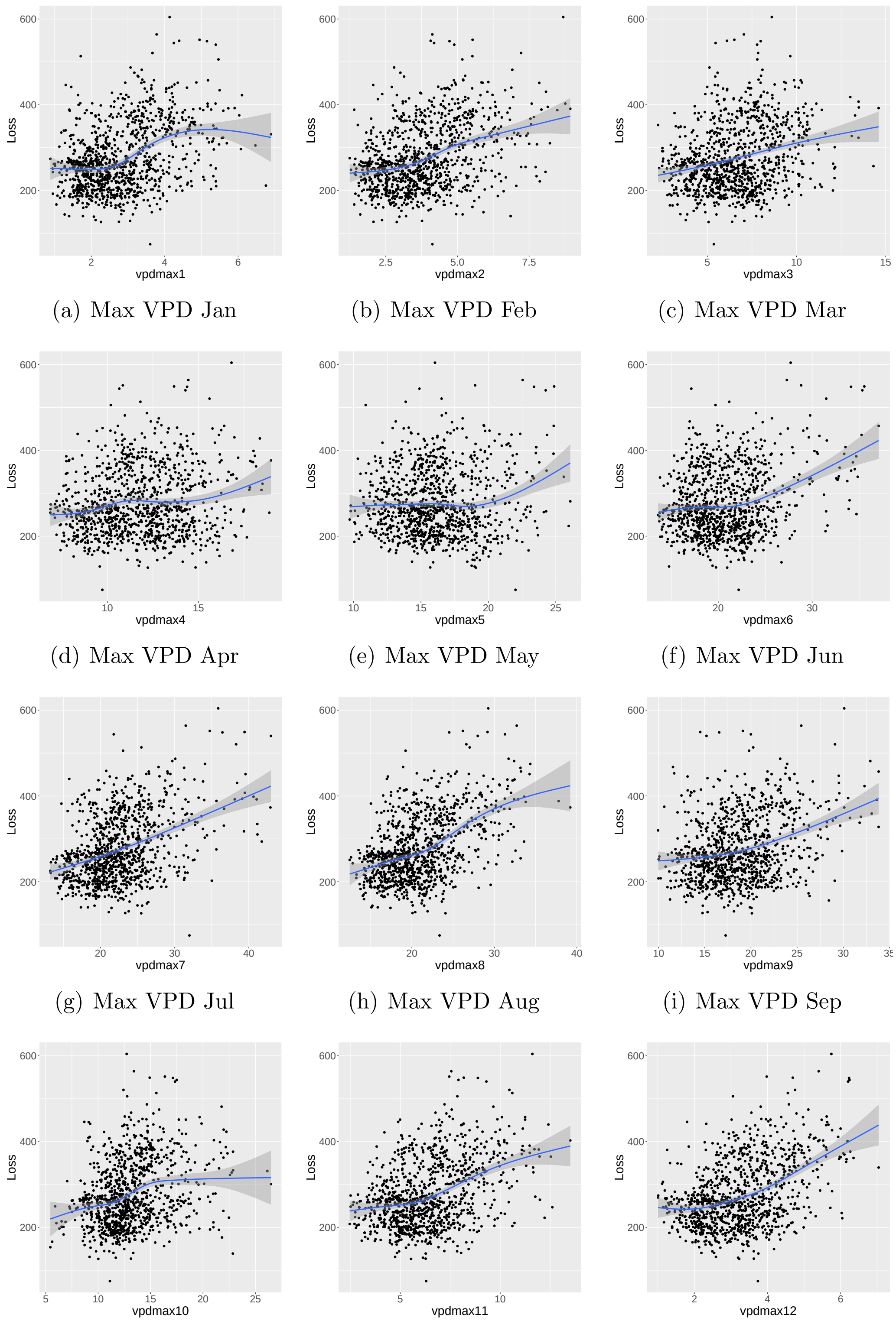


(k) Min Temperature Nov



(l) Min Temperature Dec

Figure F.4: Scatterplots of minimum temperature (Jan-Dec) with crop losses.



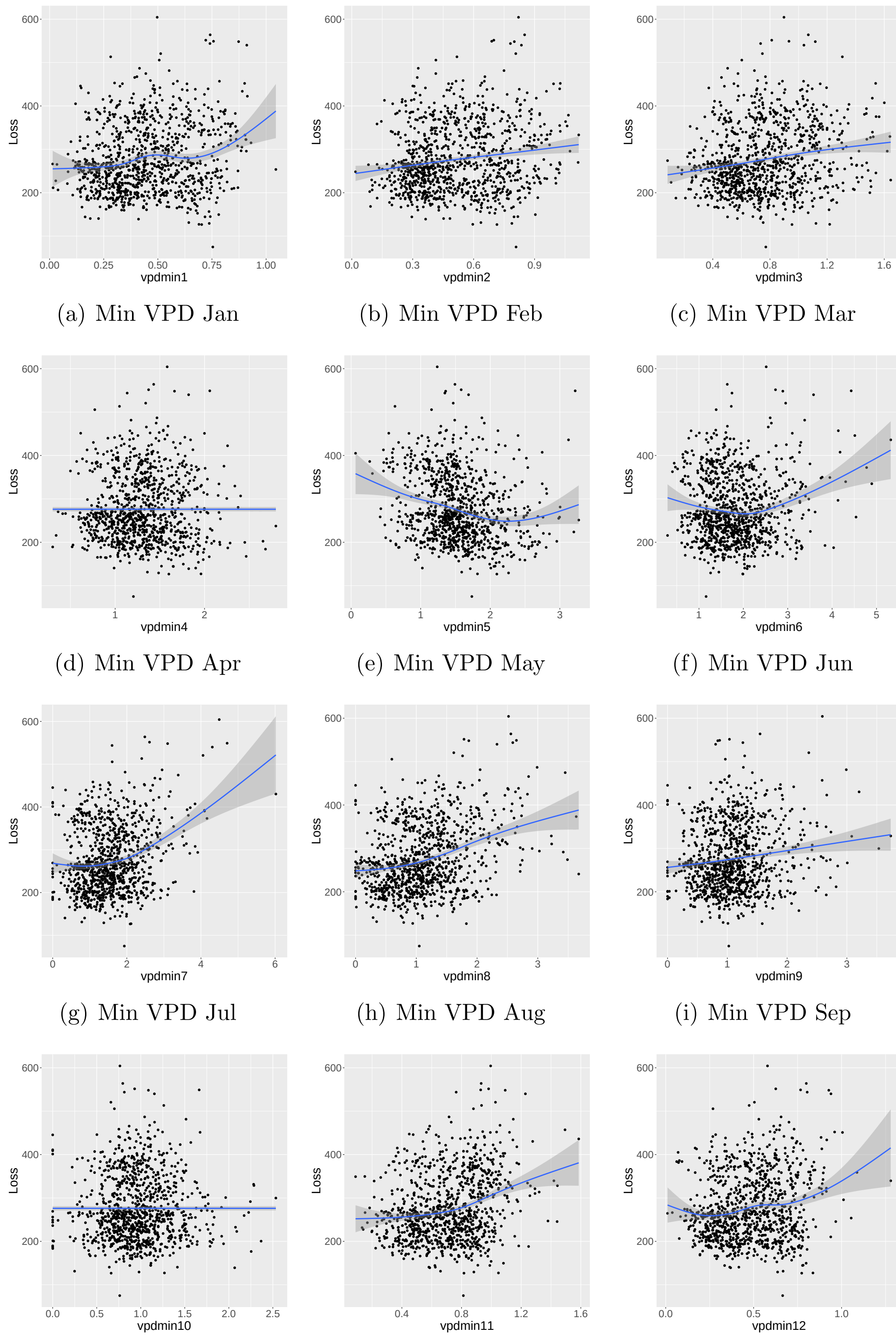
(a) Max VPD Jan (b) Max VPD Feb (c) Max VPD Mar

(d) Max VPD Apr (e) Max VPD May (f) Max VPD Jun

(g) Max VPD Jul (h) Max VPD Aug (i) Max VPD Sep

(j) Max VPD Oct (k) Max VPD Nov (l) Max VPD Dec

Figure F.5: Scatterplots of maximum vapor pressure deficit (Jan-Dec) with crop losses.



(a) Min VPD Jan
(b) Min VPD Feb
(c) Min VPD Mar
(d) Min VPD Apr
(e) Min VPD May
(f) Min VPD Jun
(g) Min VPD Jul
(h) Min VPD Aug
(i) Min VPD Sep
(j) Min VPD Oct
(k) Min VPD Nov
(l) Min VPD Dec

Figure F.6: Scatterplots of minimum vapor pressure deficit (Jan-Dec) with crop losses.

G Data homogeneity

Agricultural risk management is often faced with the challenge of data scarcity, since crop yield data are only recorded at annual frequency. Therefore, in order to increase the data sample size, and hence the performance of the trained NN-based index insurance, we assume in this paper that crop yield losses are both time and space homogeneous and expand the sample size to 7,869 county-years. In this appendix, we verify the data homogeneity assumptions. In order to guarantee the time homogeneity, following the literature, we perform a series of statistical analysis to remove trends and heteroscedasticity in the data (see Section 4.1.1 for details). The time homogeneity of our detrended data could be justified by the similarity of data in the three disjoint samples. For example, the utility without insurance in the training, validation, and test samples are -3.99, -3.99, and -4.16, respectively, which are very close. For spatial homogeneity, we perform a simple test by inspecting the homogeneity assumption in the results. In particular, we randomly combine two counties into one location and train the NN-based index insurance again. The results are displayed in Table G.1. We can see the results of the NN-based insurance trained with the randomly combined sample are similar to those trained with the original sample in our main analysis (the performance in the randomly combined sample declines slightly compared to the baseline results because of the reduced sample size). This quantitatively confirms the spatial homogeneity assumption.

Table G.1: **Data homogeneity.** We validate the data homogeneity assumption. In particular, we randomly combine two counties into one location and train the NN-based index insurance again. Panel A summarizes utilities with and without (w/o) different index insurance policies and the percentage of utility improvement. Panel B summarizes CEW with and without (w/o) index insurance policies and certainty equivalent wealth (CEW) improvements in dollars and as a percentage. Panel C summarizes policy characteristics, including premiums, coverage, and profits for the insurers. Panel D summarizes the risk reduction effect of different index insurance policies, measured by the standard deviation of wealth. Panel E summarizes the risk reduction at the tail, measured by the 5%-level value-at-risk (VaR). "BL" represents the baseline case studied in Section 4.2. The risk loading parameter at equilibrium (λ^*) for each contract is reported in parentheses.

Sample	Original sample (BL) ($\lambda^* = 1.2414$)		Randomly combined sample ($\lambda^* = 1.2329$)	
Data	Training	Test	Training	Test
Panel A: Utility improvement				
U with insurance	-3.57	-3.57	-3.65	-3.69
U w/o insurance	-3.99	-4.16	-4.07	-4.23
U improvement (%)	10.60%	14.35%	10.51%	12.76%
Panel B: CEW improvement				
CEW with insurance	444.64	444.61	441.85	440.20
CEW w/o insurance	430.63	425.26	427.97	423.14
CEW improvement	14.00	19.36	13.88	17.06
CEW improvement (%)	3.25%	4.55%	3.24%	4.03%
Panel C: Policy characteristics				
Premium	28.44	28.72	29.66	29.08
Coverage	22.91	23.13	24.06	23.59
Insurer Profit	5.53	5.59	5.60	5.49
Panel D: Risk reduction measured by standard deviation				
Std	54.05	47.49	46.07	42.26
Std w/o insurance	81.94	78.92	76.73	74.03
Std reduction	34.04%	39.82%	39.96%	42.92%
Panel E: Risk reduction measured by Value-at-Risk (VaR)				
$\text{VaR}_{5\%}$	382.89	379.64	392.84	390.32
$\text{VaR}_{5\%}$ w/o insurance	316.28	325.91	320.35	323.69
$\text{VaR}_{5\%}$ improvement	66.61	53.73	72.49	66.63

H Ranking weather indices according to gradient-based sensitivities

This appendix displays ranking of all weather indices based on their gradient-based sensitivities to insurance payoffs from the NN-based index insurance contract and their absolute correlations with production losses, in Figure H.1. The number on top of each bar is the rank difference between using the sensitivity analysis and the absolute correlation. We can see from Figure H.1 that some weather indices are impactful in terms of both absolute correlation and sensitivities (those ranked high with small rank differences, e.g., tmax12, vpdmax8), whereas some weather indices are impactful based on sensitivities but not correlations (those ranked high with larger rank differences, e.g., dpt11, vpdmin5). From the perspective of designing effective index insurance contracts, those weather indices with large absolute value of correlations are not necessarily the most important ones.

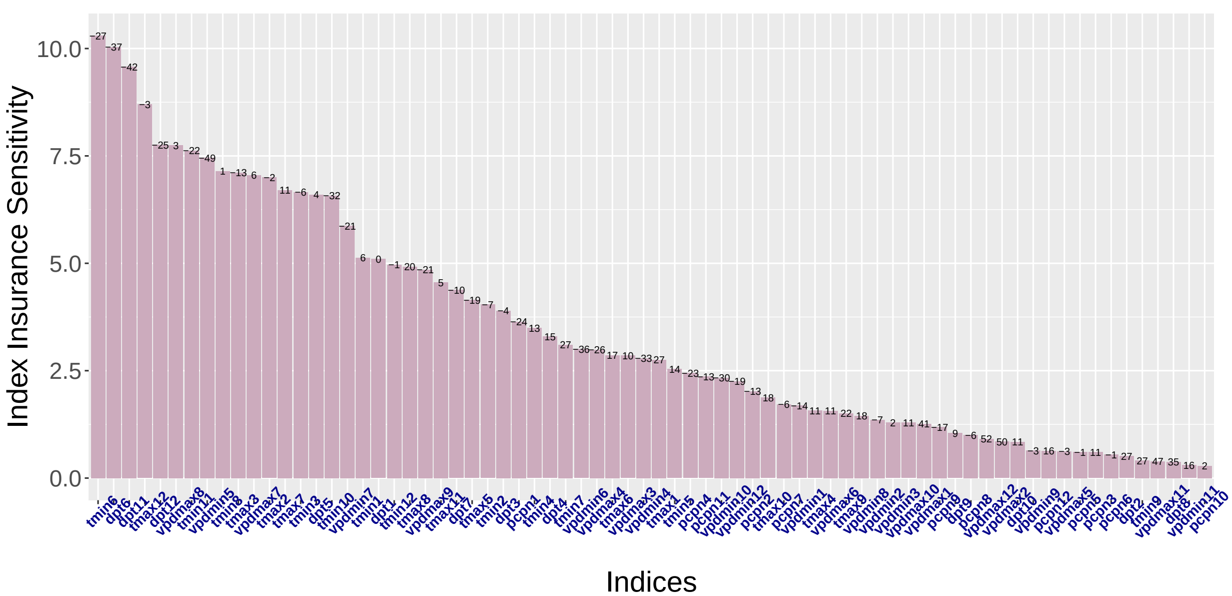
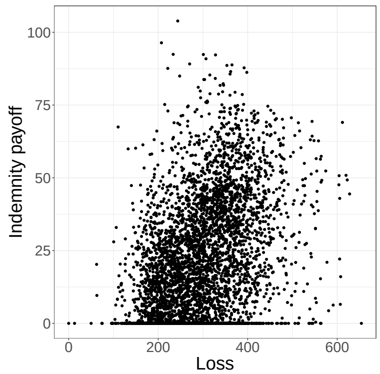


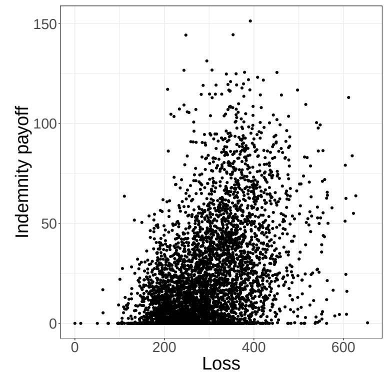
Figure H.1: **Rankings of indices according to index insurance sensitivities.** The number on top of each bar is the rank difference between using the sensitivity analysis criterion and the absolute correlation criterion.

I Basis risk of 7 index insurance contracts

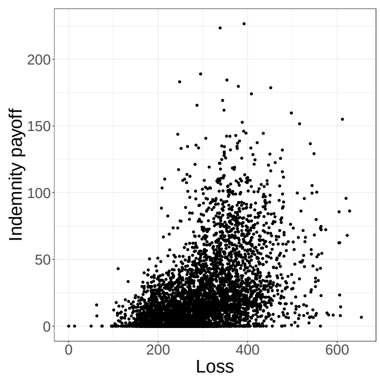
In this subsection, we compare basis risk of seven index insurances considered in Section 4.4. Panel (a) of Figure 1 replicates the large basis risk observed in current practice, which is a single-index piecewise-linear insurance contract (*Linear1*). Figures I.1 and I.2 illustrate how well insurance payoffs match the real losses incurred for the other six index insurance contracts discussed above, using the training sample and test sample, respectively. Across all contracts, except *NN72*, we observe a notably large mismatch between losses and insurance payoffs, especially for the test set. In contrast, *NN72* has a payoff function that is similar to the stop-loss payoff function of a conventional indemnity-based insurance, indicating its dramatic accuracy in mimicking the actual losses by utilizing complex information conveyed in the weather variables. Therefore, the baseline model achieves low basis risk, which is similar to a conventional indemnity-based insurance. These results illustrate the importance of using nonlinear, high-dimensional inputs when designing the index insurance contracts.



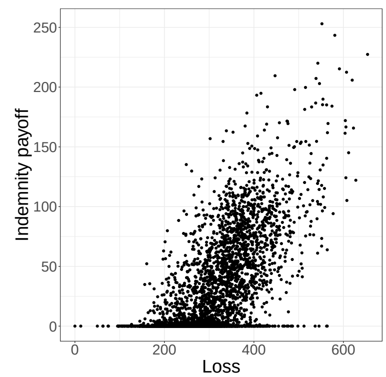
(a) Linear5



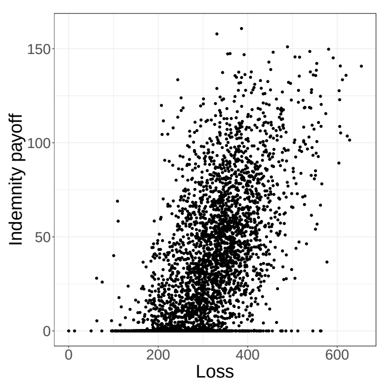
(b) Quadratic5



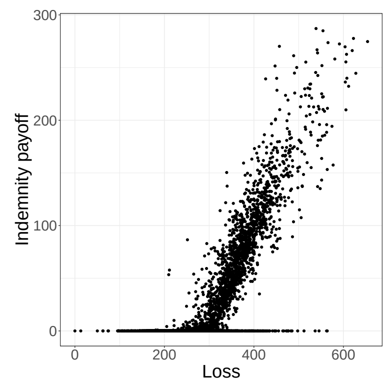
(c) Cubic5



(d) NN5

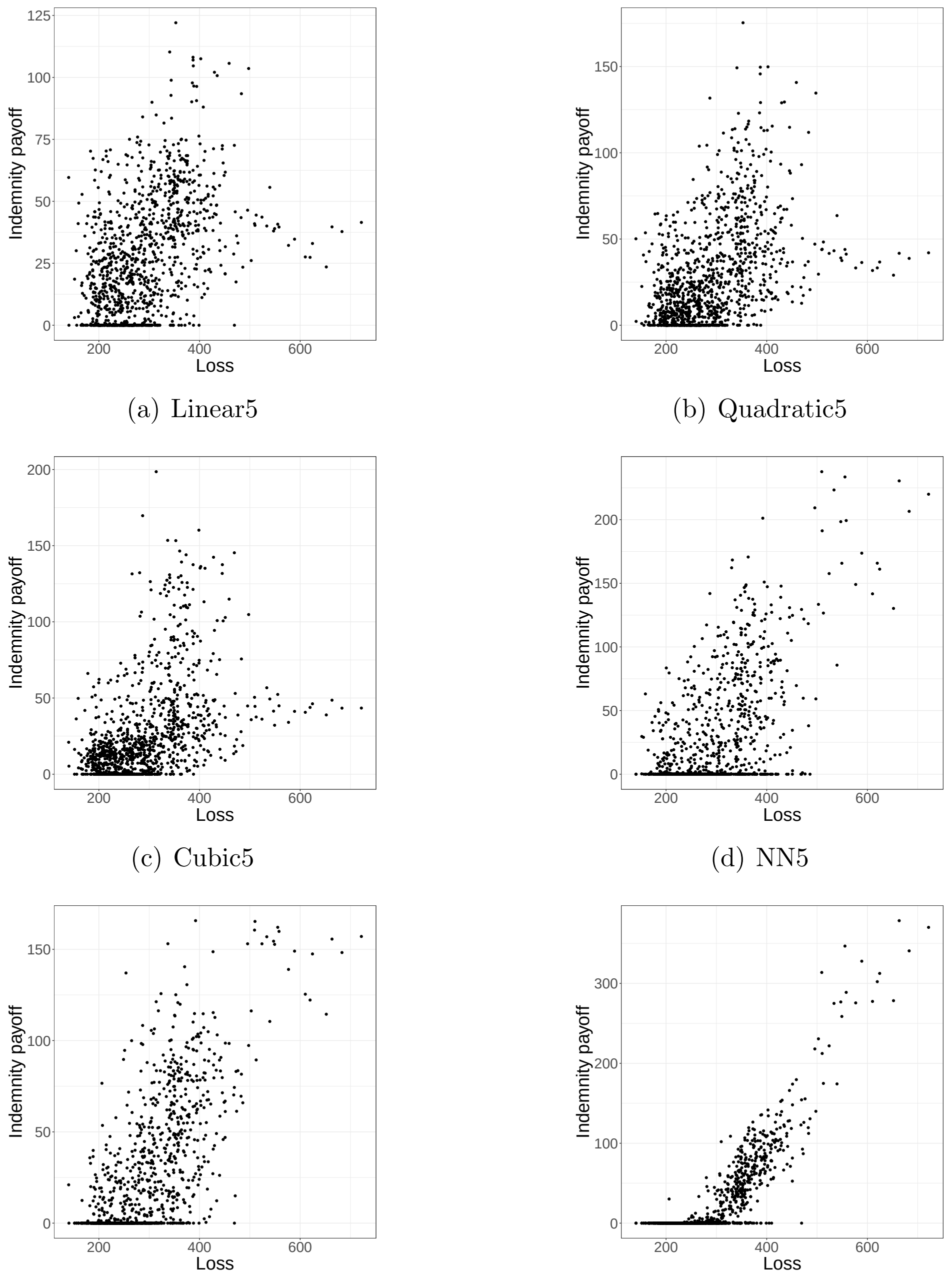


(e) Linear72



(f) NN72

Figure I.1: **Basis risk of various index insurance contracts, using the training sample.** These panels plot the insurance payoffs against actual loss, using the training sample. Six insurance contracts are presented, including (a) a linear insurance contract with five weather indices (Linear5); (b) a quadratic insurance contract with five weather indices (Quadratic5); (c) a cubic insurance contract with five weather indices (Cubic5); (d) an NN-based contract with five weather indices (NN5); (e) a linear insurance contract with 72 weather indices (Linear72); and (f) the baseline model (NN72, an NN-based contract with 72 weather indices).



(a) Linear5
(b) Quadratic5
(c) Cubic5
(d) NN5
(e) Linear72
(f) NN72

Figure I.2: **Basis risk of various index insurance contracts, using the test sample.** These panels plot the insurance payoffs against actual loss, using the test sample. Six insurance contracts are presented, including (a) a linear insurance contract with five weather indices (Linear5); (b) a quadratic insurance contract with five weather indices (Quadratic5); (c) a cubic insurance contract with five weather indices (Cubic5); (d) an NN-based contract with five weather indices (NN5); (e) a linear insurance contract with 72 weather indices (Linear72); and (f) the baseline model (NN72, an NN-based contract with 72 weather indices).

J The impacts of dimensionality

In this subsection, we investigate the impact of dimensionality on the NN-based index insurance performance. We consider models with the most important 1, 18, 36, 54, and 72 weather indices. The index importance is ranked based on the gradient-based sensitivity analysis discussed in Section 4.3. We see that using only one index improves the utility by 0.47% in the test set. Adding more weather indices significantly improves the model performances. For example, the model with 36 weather indices improves the utility by 13.40% in the test set. This analysis demonstrates the importance of including higher dimensional inputs in the NN-based index insurance contract.

Table J.1: **Comparing models with various number of weather indices.** We evaluate the performance of the NN-based index insurance with different number of weather indices in the test set, using 1, 18, 36, 54, and 72 weather indices. Panel A summarizes utilities with and without (w/o) different index insurance policies and the percentage of utility improvement. Panel B summarizes CEW with and without (w/o) index insurance policies and certainty equivalent wealth (CEW) improvements in dollars and as a percentage. Panel C summarizes policy characteristics, including premiums, coverage, and profits for the insurers. Panel D summarizes the risk reduction effect of different index insurance policies, measured by the standard deviation of wealth. Panel E summarizes the risk reduction at the tail, measured by the 5%-level value-at-risk (VaR). "BL" represents the baseline case studied in Section 4.2.

	72 indices (BL)	54 indices	36 indices	18 indices	One index
Panel A: Utility improvement					
U with insurance	-3.57	-3.60	-3.61	-3.67	-4.14
U w/o insurance	-4.16	-4.16	-4.16	-4.16	-4.16
U improvement (%)	14.35%	13.64%	13.40%	11.90%	0.47%
Panel B: CEW improvement					
CEW with insurance	444.61	443.58	443.24	441.10	425.84
CEW w/o insurance	425.26	425.26	425.26	425.26	425.26
CEW improvement	19.36	18.33	17.99	15.84	0.58
CEW improvement (%)	4.55%	4.31%	4.23%	3.73%	0.14%
Panel C: Policy characteristics					
Premium	28.72	28.56	27.12	21.61	27.24
Coverage	23.13	23.01	21.85	17.41	25.91
Insurer Profit	5.59	5.55	5.27	4.20	1.33
Panel D: Risk reduction measured by standard deviation					
Std	47.49	49.46	50.94	56.63	72.92
Std w/o insurance	78.92	78.92	78.92	78.92	78.92
Std reduction	39.82%	37.33%	35.45%	28.24%	7.60%
Panel E: Risk reduction measured by Value-at-Risk (VaR)					
$\text{VaR}_{5\%}$	379.64	371.54	370.77	357.98	335.30
$\text{VaR}_{5\%}$ w/o insurance	325.91	325.91	325.91	325.91	325.91
$\text{VaR}_{5\%}$ improvement	53.73	45.64	44.86	32.08	9.39

K Weather predictability

In the past four decades, numerical weather prediction technology has been improving a lot. That is, adding one day of predictive power per decade (Bauer et al. 2015). However, long-term (e.g., several months or one-year ahead) weather is still unpredictable (Alley et al. 2019, Voosen 2019). For example, Figure K.1 plots the forecast skill at three-, five-, seven-, and ten-day ranges. The best forecast at the European Centre for Medium-Range Weather Forecasts (ECMWF) runs out to around 10 days. In fact, research shows that there indeed exists a predictability limit for weather forecast, which is 4-5 days in general and 10 days for midlatitude weather.

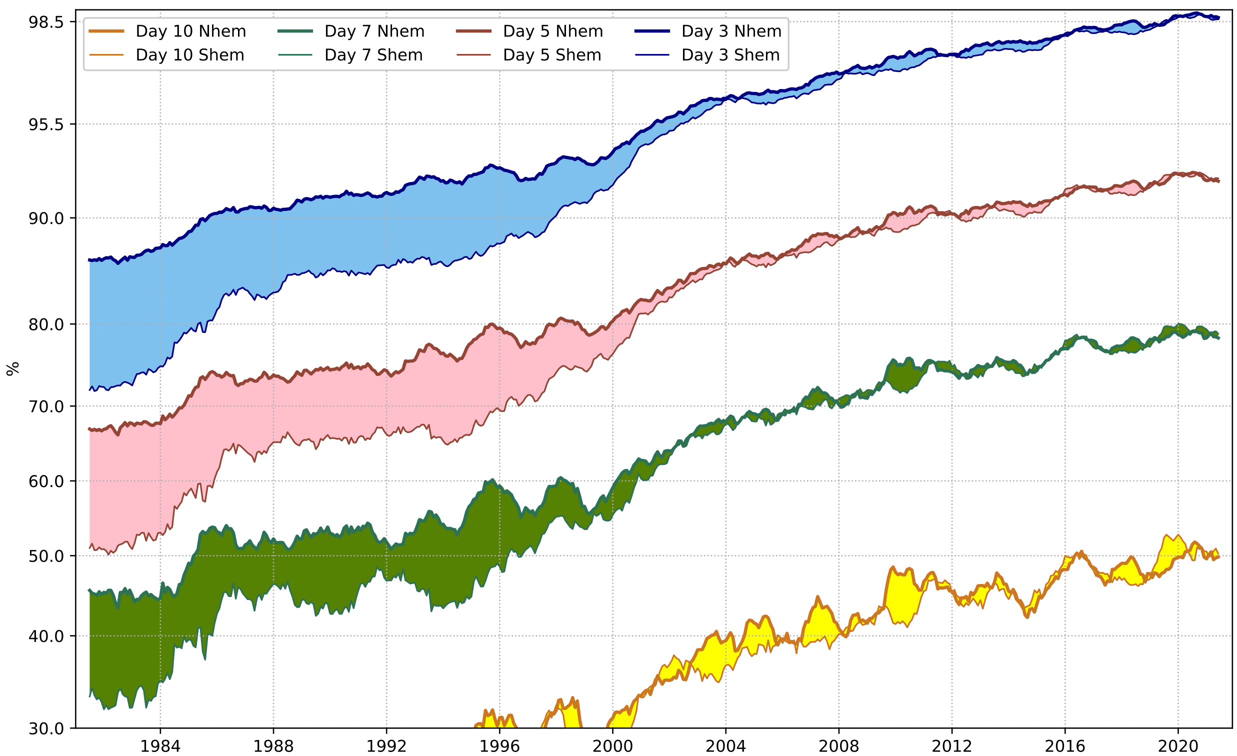


Figure K.1: **The evolution of weather forecast quality.** This figure plots the forecast skill at three-, five-, seven-, and ten-day ranges. The forecast skill is measured by anomaly correlation coefficient (ACC) of the height of 500-hPa level between the forecasts and observations. The two curves are computed over the extra-tropical northern and southern hemispheres. In practice, a value higher than 60% is treated as a skillful weather forecast. This plot is adapted from the ECMWF official website (`https://www.ecmwf.int/en/forecasts/charts/catalogue/plwww_m_hr_ccaf_adrian_ts`).

L Robustness checks

We further perform several robustness checks in this section. First, we examine the impacts of insurers' characteristics. We consider various insurers' supply curves and also exogenously given risk loading. Second, we examine the impacts of farmers' characteristics, such as different coverage levels, risk aversion, and alternative utility functions (e.g., log utility and power utility) on the NN-based index insurance contract.

L.1 Different insurers' supply curves

In our previous analysis, the equilibrium loading parameter, λ^*, was determined via a reduced-form approach, where the supply curve is estimated using market data from the USDA SOB Reports, which might have simultaneity issue. To address these concerns, in this subsection, we further investigate the robustness of insurers' supply curve. We use the upper and lower bounds of [10%, 90%] and [25%, 75%] confidence intervals of the supply curve estimates to determine the equilibrium loading parameter. Figure L.1 displays our estimated supply curve with its confidence intervals.

Table L.1 summarizes the results. When insurers' supply curve shifts to the upper bounds of its confidence intervals, equilibrium loading parameter, λ^*, decreases, and the equilibrium insurance demand increases. As a consequence, farmers buy more index insurance, and achieve more utility improvements and higher CEW. While insurance demand increases, we observe that insurers do not gain higher profits as the insurance is priced lower. When the insurance supply curve moves toward the lower bounds of its confidence intervals, λ^* increases and insurance demand decreases. Correspondingly, farmers' utilities and CEW improvements are reduced. Overall, under various supply curves, the NN-based index insurance contract provides robust results for utility and CEW improvements, risk reduction, and insurers' profits. Therefore, potential simultaneity issue barely affects the results.

L.2 Exogenous risk loading and market demand

Previously, we mainly analyze the equilibrium when insurance premium is endogenously determined. However, it is possible that there are some exogenous sources affecting the insurance premium. For example, supply frictions such as administration costs, litigation risks and regulatory frictions that insurers might face, or imperfect competition among insurers. This can be captured by a partial equilibrium case where insurers can choose risk loading, λ. To this end, we consider exogenous values of $\lambda = 1.33, 1.37$, and 1.415, which corresponds to the case when insurance demand reduces by $20\%, 30\%$, and 40% relative to the endogenous case, respectively. The results are summarized in Table L.2. We see that as the insurance become more expensive, both the farmer's incentive to purchase insurance and utility improvement decrease. However, it is important to note that even with the largest demand reduction of 40%, the policyholder gains a utility improvement of 11.31% and a CEW improvement of $15 in the test sample, and basis risk is significantly reduced, as measured by either standard deviation or VaR. Finally, the insurer is observed to have a trade-off when determining its risk loading. While a larger λ leads to a higher profit margin, it negatively affects its market demand and thus the total profit.

L.3 Farmers with different coverage demands

In the baseline case, we focus on farmers who are solely interested in maximizing their utility, regardless of the coverage they purchase and premiums they pay. In practice, however, farmers often have a predetermined level of coverage in mind, because of either a better understanding of their financial position and insurance demand, or a relatively tight budget constraint. As a result, these farmers may be interested in more customized index insurance contracts. The NN-based index insurance design proposed in this paper is convenient to create customized contracts to meet their demands.

For illustration purposes, we consider a set of index insurance plans with a coverage of $10, $20, $30, and $40. Table L.3 summarizes the results of these four contracts. For comparison purposes, we also list the baseline model which has the optimal coverage of \$23.13. We see that the amount of utility improvement first increases with coverage, peaking in the baseline case, and then decreasing with the coverage level. Overall, the NN-based insurance contract provides reasonable utility improvement for various coverage levels.

L.4 Farmers with different levels of risk aversion

Farmers' risk aversion varies with their age, education, farming experience, wealth, etc. One might wonder how different risk appetites lead to different demands for insurance. In this subsection, we consider policyholders with various levels of risk aversion. In addition to the baseline case in which $\alpha = 0.008$ (corresponding to a relative risk aversion of 3.1), we consider farmers with relative risk aversions of $2, 4$, and 5, which correspond to absolute risk aversion coefficients of $\alpha = 0.0051, 0.0103$, and 0.0129, respectively. Table L.4 summarizes the results. Table L.4 shows that the optimal index insurance design achieves greater utility and CEW improvements for farmers with higher risk aversion. This is because more risk-averse policyholders are more concerned about volatilities in their wealth. As such, insurers can charge these farmers higher premiums (i.e., imposing a higher loading parameter, λ^*).

Next, we consider farmers with time-varying risk aversion which depends on losses in the previous year. For example, farmers might become more risk averse after large losses, especially for less educated farmers without long-term learning skills (Cai et al. 2020). Such time-varying risk aversion might capture time inconsistency as well. Specifically, we first compute the 75^{th} and 25^{th} percentiles of yield loss. Suppose the farmer's average relative risk aversion is $RRA = 3.1$ (absolute risk aversion is $\alpha = 0.008$). If the farmer experiences a loss larger than the 75^{th} percentile in year $t-1$, her risk aversion in year t becomes $3.1 \times (1+x)$; on the contrary, if the farmer experiences a loss lower than the 25^{th} percentile in year $t-1$, her risk aversion in year t is $3.1 \times (1-x)$. That is, the farmer's risk aversion is $3.1 \times (1-x), 3.1$, or $3.1 \times (1+x)$, depending on the previous loss experience. We consider different levels of risk aversion variations, i.e., $x = 0.1, 0.2$, and 0.3. The results are summarized in Table L.5.

Generally, we see that the NN-based index insurance consistently improves farmers' utility and CEW, and reduces basis risk across the specifications. Nevertheless, time-varying risk aversion impedes the performance of the designed index insurance, with larger variations of risk aversion hindering the insurance performance more.

L.5 Alternative utility functions

In this subsection, we evaluate the performance of the proposed NN-based index insurance using constant relative risk aversion (CRRA) utility functions. We consider the power utility with various levels of risk aversion, that is, relative risk aversion (RRA) of 2, 3, 4, and 5, and log utility (RRA = 1). Again, we use the 3-hidden-layer (64-64-16 neurons) structure, as in the baseline model. Table L.6 summarizes the results.[2] The performance is similar to the baseline case with negative exponential utility. Using log utility, we find that the farmer's utility and CEW improvements are marginal because the risk aversion of her log utility is low (RRA = 1). Evaluating the results of power utility, we see that as policyholders become more risk averse, they purchase more coverage, and insurers also make higher profits. Index insurance performance also significantly increases with risk aversion. For example, the CEW improvement for RRA = 5 is about five times larger than that for RRA = 2. Comparing the results with the negative exponential utility case in Table L.4, we observe that the insurance has higher CEW improvements, given the same RRA. This is because power utility functions penalize extremely low wealth cases more severely.

[2] Logarithm utility and power utility functions are not defined for negative wealth. Therefore, to avoid the negative wealth cases, we winsorize the loss data at 99% percentile.

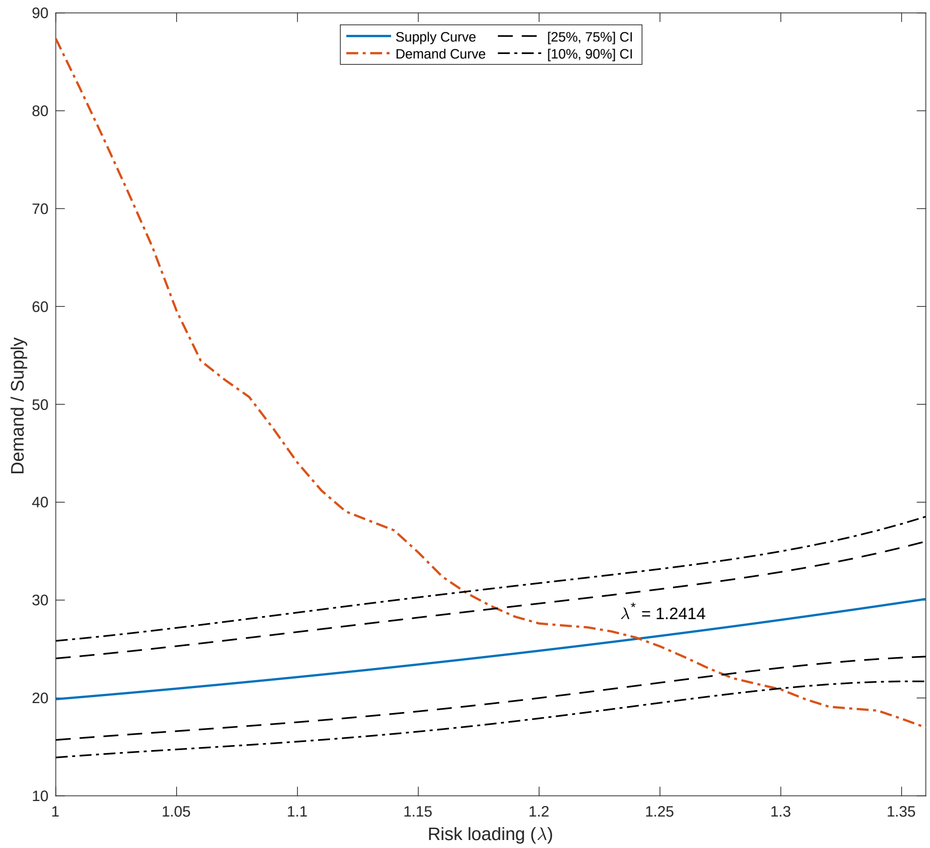


Figure L.1: **Confidence intervals of the insurance supply curve.** This figure displays the upper and lower bounds of the [10%, 90%] and [25%, 75%] confidence intervals for the estimated supply curve of the index insurance. The insurance supply curve is fitted from the USDA SOB Reports data with a power function using the nonlinear least squares method. The demand curve is for the NN-based optimal index insurance with a 3-hidden-layer (64-64-16 neurons) structure, and is fitted with a piecewise cubic hermite interpolating polynomial.

Table L.1: **Impacts of insurer's supply curves.** We test the robustness of our results using the upper and lower bounds of [10%, 90%] and [25%, 75%] confidence intervals (CI) of the supply curve estimates. Panel A summarizes utilities with and without (w/o) index insurance policies and the percentage of utility improvement. Panel B summarizes certainty equivalent wealth (CEW) with and without (w/o) index insurance and the CEW improvements in dollars and as a percentage. Panel C summarizes policy characteristics, including premiums, coverage, and profits for the insurer. Panel D summarizes the risk reduction effect of different index insurance policies, measured by the standard deviation of wealth. Panel E summarizes the risk reduction at the tail, measured by the 5%-level value-at-risk (VaR). The risk loading parameter at equilibrium (λ^*) for each contract is reported in parentheses.

Supply curve	[25%, 75%] CI				[10%, 90%] CI			
	Lower bound		Upper bound		Lower bound		Upper bound	
	($\lambda^* = 1.2814$)		($\lambda^* = 1.1871$)		($\lambda^* = 1.3142$)		($\lambda^* = 1.1744$)	
Data	Training	Test	Training	Test	Training	Test	Training	Test
Panel A: Utility improvement								
U with insurance	-3.57	-3.65	-3.53	-3.53	-3.59	-3.67	-3.52	-3.52
U w/o insurance	-3.99	-4.16	-3.99	-4.16	-3.99	-4.16	-3.99	-4.16
U improvement (%)	10.43%	12.34%	11.53%	15.31%	9.95%	11.81%	11.77%	15.54%
Panel B: CEW improvement								
CEW with insurance	444.41	441.72	445.95	446.03	443.73	440.96	446.28	446.37
CEW w/o insurance	430.63	425.26	430.63	425.26	430.63	425.26	430.63	425.26
CEW improvement	13.77	16.46	15.32	20.77	13.10	15.70	15.65	21.11
CEW improvement (%)	3.20%	3.87%	3.56%	4.89%	3.04%	3.69%	3.63%	4.96%
Panel C: Policy characteristics								
Premium	26.28	26.84	29.38	30.07	26.57	25.92	31.38	31.73
Coverage	20.51	20.95	24.75	25.33	20.21	19.72	26.72	27.02
Insurer Profit	5.77	5.90	4.63	4.74	6.35	6.20	4.66	4.71
Panel D: Risk reduction measured by standard deviation								
Std	54.82	53.27	52.86	45.89	55.06	54.09	51.78	44.94
Std w/o insurance	81.94	78.92	81.94	78.92	81.94	78.92	81.94	78.92
Std reduction	33.10%	32.49%	35.49%	41.85%	32.81%	31.46%	36.81%	43.06%
Panel E: Risk reduction measured by Value-at-Risk (VaR)								
$\mathrm{VaR}_{5\%}$	384.10	362.39	384.30	383.05	383.67	358.27	385.54	384.74
$\mathrm{VaR}_{5\%}$ w/o insurance	316.28	325.91	316.28	325.91	316.28	325.91	316.28	325.91
$\mathrm{VaR}_{5\%}$ improvement	67.82	36.49	68.03	57.14	67.39	32.36	69.27	58.83

Table L.2: **Exogenously specified risk loading.** This table compares insurance contract performances when risk loading is exogenously specified. We consider some exogenously given risk loadings: $\lambda^* = 1.33, 1.37$ and 1.415, which corresponds to a reduction of demand by 20%, 30%, and 40%, relative to the baseline model. Panel A summarizes utilities with and without (w/o) different index insurance policies and the percentage of utility improvement. Panel B summarizes CEW with and without (w/o) index insurance policies and certainty equivalent wealth (CEW) improvements in dollars and as a percentage. Panel C summarizes policy characteristics, including premiums, coverage, and profits for the insurers. Panel D summarizes the risk reduction effect of different index insurance policies, measured by the standard deviation of wealth. Panel E summarizes the risk reduction at the tail, measured by the 5%-level value-at-risk (VaR). "BL" represents the baseline case studied in Section 4.2.

Coverage reduction	0% (BL)		20%		30%		40%	
Risk loading	$\lambda = 1.2414$ (BL)		$\lambda = 1.33$		$\lambda = 1.37$		$\lambda = 1.415$	
Data	Training	Test	Training	Test	Training	Test	Training	Test
Panel A: Utility improvement								
U with insurance	-3.57	-3.57	-3.63	-3.64	-3.65	-3.68	-3.68	-3.69
U w/o insurance	-3.99	-4.16	-3.99	-4.16	-3.99	-4.16	-3.99	-4.16
U improvement (%)	10.60%	14.35%	9.03%	12.60%	8.39%	11.60%	7.84%	11.31%
Panel B: CEW improvement								
CEW with insurance	444.64	444.61	442.46	442.10	441.58	440.67	440.84	440.26
CEW w/o insurance	430.63	425.26	430.63	425.26	430.63	425.26	430.63	425.26
CEW improvement	14.00	19.36	11.82	16.84	10.95	15.41	10.21	15.00
CEW improvement (%)	3.25%	4.55%	2.75%	3.96%	2.54%	3.62%	2.37%	3.53%
Panel C: Policy characteristics								
Premium	28.44	28.72	24.43	24.37	22.06	20.02	19.33	17.18
Coverage	22.91	23.13	18.37	18.32	16.10	14.61	13.66	12.14
Insurer Profit	5.53	5.59	6.06	6.05	5.96	5.41	5.67	5.04
Panel D: Risk reduction measured by standard deviation								
Std	54.05	47.49	57.50	52.16	59.64	56.42	61.88	58.30
Std w/o insurance	81.94	78.92	81.94	78.92	81.94	78.92	81.94	78.92
Std reduction	34.04%	39.82%	29.83%	33.90%	27.22%	28.50%	24.48%	26.13%
Panel E: Risk reduction measured by Value-at-Risk (VaR)								
$\text{VaR}_{5\%}$	382.89	379.64	379.56	371.51	373.37	362.77	369.05	362.05
$\text{VaR}_{5\%}$ w/o insurance	316.28	325.91	316.28	325.91	316.28	325.91	316.28	325.91
$\text{VaR}_{5\%}$ improvement	66.61	53.73	63.28	45.61	57.09	36.86	52.78	36.14

Table L.3: **Impacts of coverage level.** We consider an NN-based index insurance with various coverage levels. Panel A summarizes utilities with and without (w/o) index insurance policies and the percentage of utility improvement. Panel B summarizes certainty equivalent wealth (CEW) with and without (w/o) index insurance policies and CEW improvements in dollars and as a percentage. Panel C summarizes the risk reduction effect of different index insurance policies, measured by the standard deviation of wealth. Panel D summarizes the risk reduction at the tail, measured by the 5%-level value-at-risk (VaR). "BL" indicates the baseline case studied in Section 4.2.

	Coverage: \$10		Coverage: \$20		Coverage: \$23.13(BL)		Coverage: \$30		Coverage: \$40	
Data	Training	Test	Training	Test	Training	Test	Training	Test	Training	Test
Panel A: Utility improvement										
U with insurance	-3.63	-3.66	-3.56	-3.58	-3.57	-3.57	-3.58	-3.58	-3.62	-3.64
U w/o insurance	-3.99	-4.16	-3.99	-4.16	-3.99	-4.16	-3.99	-4.16	-3.99	-4.16
U improvement (%)	8.98%	12.02%	10.60%	14.11%	10.60%	14.35%	10.34%	13.95%	9.21%	12.54%
Panel B: CEW improvement										
CEW with insurance	442.40	441.26	444.65	444.28	444.64	444.61	444.28	444.04	442.71	442.00
CEW w/o insurance	430.63	425.26	430.63	425.26	430.63	425.26	430.63	425.26	430.63	425.26
CEW improvement	11.76	16.00	14.01	19.02	14.00	19.36	13.65	18.78	12.07	16.75
CEW improvement (%)	2.73%	3.76%	3.25%	4.47%	3.25%	4.55%	3.17%	4.42%	2.80%	3.94%
Panel C: Risk reduction measured by standard deviation										
Std	65.61	61.72	56.03	50.21	54.05	47.49	50.71	43.94	47.97	42.08
Std w/o insurance	81.94	78.92	81.94	78.92	81.94	78.92	81.94	78.92	81.94	78.92
Std reduction	19.94%	21.79%	31.62%	36.37%	34.04%	39.82%	38.11%	44.32%	41.46%	46.68%
Panel D: Risk reduction measured by Value-at-Risk (VaR)										
$\text{VaR}_{5\%}$	359.65	353.98	380.75	374.99	382.89	379.64	383.57	382.84	380.75	379.06
$\text{VaR}_{5\%}$ w/o insurance	316.28	325.91	316.28	325.91	316.28	325.91	316.28	325.91	316.28	325.91
$\text{VaR}_{5\%}$ improvement	43.37	28.07	64.47	49.09	66.61	53.73	67.29	56.93	64.47	53.16

Table L.4: **Impacts of risk aversion.** We consider an NN-based index insurance for farmers with various levels of risk aversion, i.e., relative risk aversion (RRA) of 2, 3.1, 4, and 5. Panel A summarizes utilities with and without (w/o) different index insurance policies and the percentage of utility improvement. Panel B summarizes CEW with and without (w/o) index insurance policies and certainty equivalent wealth (CEW) improvements in dollars and as a percentage. Panel C summarizes policy characteristics, including premiums, coverage, and profits for the insurers. Panel D summarizes the risk reduction effect of different index insurance policies, measured by the standard deviation of wealth. Panel E summarizes the risk reduction at the tail, measured by the 5%-level value-at-risk (VaR). "BL" represents the baseline case studied in Section 4.2. The risk loading parameter at equilibrium (λ^*) for each contract is reported in parentheses.

Risk aversion	$\alpha = 0.0051$ (RRA $= 2$) ($\lambda^* = 1.1753$)		$\alpha = 0.008$ (BL, RRA $= 3.1$) ($\lambda^* = 1.2414$)		$\alpha = 0.0103$ (RRA $= 4$) ($\lambda^* = 1.2744$)		$\alpha = 0.0129$ (RRA $= 5$) ($\lambda^* = 1.2876$)	
Data	Training	Test	Training	Test	Training	Test	Training	Test
Panel A: Utility improvement								
U with insurance	-18.47	-18.64	-3.57	-3.57	-1.04	-1.04	-0.28	-0.28
U w/o insurance	-19.23	-19.53	-3.99	-4.16	-1.29	-1.42	-0.41	-0.51
U improvement (%)	3.96%	4.57%	10.60%	14.35%	19.13%	26.94%	31.37%	45.87%
Panel B: CEW improvement								
CEW with insurance	450.60	448.79	444.64	444.61	440.20	440.63	435.24	436.75
CEW w/o insurance	442.84	439.80	430.63	425.26	419.59	410.16	406.06	389.18
CEW improvement	7.76	8.99	14.00	19.36	20.61	30.47	29.18	47.58
CEW improvement (%)	1.75%	2.05%	3.25%	4.55%	4.91%	7.43%	7.19%	12.22%
Panel C: Policy characteristics								
Premium	25.34	24.14	28.44	28.72	29.56	29.42	30.65	31.31
Coverage	21.56	20.54	22.91	23.13	23.20	23.09	23.80	24.32
Insurer Profit	3.78	3.60	5.53	5.59	6.36	6.33	6.85	6.99
Panel D: Risk reduction measured by standard deviation								
Std	54.63	51.10	54.05	47.49	53.69	48.31	53.33	48.12
Std w/o insurance	81.94	78.92	81.94	78.92	81.94	78.92	81.94	78.92
Std reduction	33.33%	35.25%	34.04%	39.82%	34.48%	38.78%	34.92%	39.03%
Panel E: Risk reduction measured by Value-at-Risk (VaR)								
$\text{VaR}_{5\%}$	385.46	372.67	382.89	379.64	382.63	374.71	382.88	371.75
$\text{VaR}_{5\%}$ w/o insurance	316.28	325.91	316.28	325.91	316.28	325.91	316.28	325.91
$\text{VaR}_{5\%}$ improvement	69.19	46.76	66.61	53.73	66.35	48.80	66.60	45.84

Table L.5: **Impacts of time-varying risk aversion.** We evaluate the index insurance performance with time-varying risk aversion. The farmer's average relative risk aversion is $RRA = 3.1$. If the farmer experiences a loss larger than the 75^{th} percentile in year $t-1$, her risk aversion in year t becomes $3.1 \times (1+x)$; on the contrary, if the farmer experiences a loss lower than the 25^{th} percentile in year $t-1$, her risk aversion in year t is $3.1 \times (1-x)$. Columns 2-7 display results for different risk aversion variations ($x = 0.1, 0.2$, and 0.3). The last two columns correspond to a constant risk aversion of 3.1, which is our baseline model. Panel A summarizes utilities with and without (w/o) different index insurance policies and the percentage of utility improvement. Panel B summarizes CEW with and without (w/o) index insurance policies and certainty equivalent wealth (CEW) improvements in dollars and as a percentage. Panel C summarizes policy characteristics, including premiums, coverage, and profits for the insurers. Panel D summarizes the risk reduction effect of different index insurance policies, measured by the standard deviation of wealth. Panel E summarizes the risk reduction at the tail, measured by the 5%-level value-at-risk (VaR).

Risk aversion	$x = 0.1$ $RRA = 2.8, 3.1, 3.4$		$x = 0.2$ $RRA = 2.5, 3.1, 3.7$		$x = 0.3$ $RRA = 2.2, 3.1, 4$		Baseline model $RRA = 3.1$	
Data	Training	Test	Training	Test	Training	Test	Training	Test
Panel A: Utility improvement								
U with insurance	-3.75	-3.79	-4.41	-4.51	-5.74	-5.94	-3.57	-3.57
U w/o insurance	-4.18	-4.33	-4.86	-4.99	-6.22	-6.33	-3.99	-4.16
U improvement (%)	10.22%	12.57%	9.23%	9.62%	7.81%	6.17%	10.60%	14.35%
Panel B: CEW improvement								
CEW with insurance	444.82	443.35	443.97	440.88	442.35	438.13	444.64	444.61
CEW w/o insurance	431.21	425.35	431.22	424.60	431.17	423.60	430.63	425.26
CEW improvement	13.61	18.00	12.75	16.27	11.18	14.54	14.00	19.36
CEW improvement (%)	3.16%	4.23%	2.96%	3.83%	2.59%	3.43%	3.25%	4.55%
Panel C: Policy characteristics								
Premium	28.24	28.56	26.05	25.83	25.35	24.60	28.44	28.72
Coverage	22.75	23.01	20.99	20.81	20.42	19.82	22.91	23.13
Insurer Profit	5.49	5.56	5.07	5.02	4.93	4.78	5.53	5.59
Panel D: Risk reduction measured by standard deviation								
Std	54.65	51.17	57.18	56.55	60.02	61.74	54.05	47.49
Std w/o insurance	81.94	78.92	81.94	78.92	81.94	78.92	81.94	78.92
Std reduction	33.30%	35.16%	30.21%	28.34%	26.76%	21.77%	34.04%	39.82%
Panel E: Risk reduction measured by Value-at-Risk (VaR)								
$\text{VaR}_{5\%}$	379.98	367.23	370.58	357.64	359.41	348.69	382.89	379.64
$\text{VaR}_{5\%}$ w/o insurance	316.59	326.59	316.59	326.59	316.59	326.59	316.28	325.91
$\text{VaR}_{5\%}$ improvement	63.39	40.64	53.99	31.05	42.82	22.10	66.61	53.73

Table L.6: **Alternative utility functions.** We consider power utility with various levels of risk aversion, i.e., relative risk aversion (RRA) of 2, 3, 4, and 5 and log utility (RRA = 1). The NN uses a 3-hidden-layer (64-64-16 neurons) structure. Panel A summarizes utilities with and without (w/o) different index insurance policies and the percentage of utility improvement. Panel B summarizes certainty equivalent wealth (CEW) with and without (w/o) index insurance policies and CEW improvements in dollars and as a percentage. Panel C summarizes policy characteristics, including premiums, coverage, and profits for the insurers. Panel D summarizes the risk reduction effect of different index insurance policies, measured by the standard deviation of wealth. Panel E summarizes the risk reduction at the tail, measured by the 5%-level value-at-risk (VaR). The risk loading parameter at equilibrium (λ^*) for each contract is reported in parentheses.

Utility function	Log Utility		Power Utility							
Risk aversion	RRA = 1 ($\lambda^* = 1.0894$)		RRA = 2 ($\lambda^* = 1.1546$)		RRA = 3 ($\lambda^* = 1.2048$)		RRA = 4 ($\lambda^* = 1.2567$)		RRA = 5 ($\lambda^* = 1.2849$)	
Data	Training	Test	Training	Test	Training	Test	Training	Test	Training	Test
Panel A: Utility improvement										
U with insurance	6.12	6.12	-2.21×10^{-03}	-2.22×10^{-03}	-2.51×10^{-06}	-2.51×10^{-06}	-3.89×10^{-09}	-3.84×10^{-09}	-7.11×10^{-12}	-6.72×10^{-12}
U w/o insurance	6.12	6.11	-2.26×10^{-03}	-2.28×10^{-03}	-2.7×10^{-06}	-2.84×10^{-06}	-4.74×10^{-09}	-5.35×10^{-09}	-10.6×10^{-12}	-13.12×10^{-12}
U improvement (%)	0.11%	0.15%	1.88%	2.70%	7.30%	11.84%	17.88%	28.25%	32.88%	48.76%
Panel B: CEW improvement										
CEW with insurance	456.89	454.70	451.87	450.47	446.76	446.59	440.89	442.75	432.97	439.11
CEW w/o insurance	453.74	450.45	443.39	438.30	430.13	419.31	412.88	396.37	391.90	371.51
CEW improvement	3.15	4.25	8.47	12.17	16.62	27.28	28.02	46.38	41.07	67.60
CEW improvement (%)	0.70%	0.94%	1.91%	2.78%	3.86%	6.51%	6.79%	11.70%	10.48%	18.20%
Panel C: Policy characteristics										
Premium	20.72	21.56	23.79	24.68	24.99	25.68	26.76	28.91	30.90	32.87
Coverage	19.02	19.79	20.60	21.37	20.74	21.32	21.29	23.00	24.05	25.58
Insurer Profit	1.70	1.77	3.19	3.31	4.25	4.37	5.47	5.91	6.85	7.29
Panel D: Risk reduction measured by standard deviation										
Std	56.79	50.38	55.59	49.18	55.53	49.03	55.17	47.52	54.37	46.03
Std w/o insurance	81.94	78.92	81.94	78.92	81.94	78.92	81.94	78.92	81.94	78.92
Std reduction	30.69%	36.17%	32.16%	37.68%	32.23%	37.88%	32.67%	39.78%	33.64%	41.67%
Panel E: Risk reduction measured by Value-at-Risk (VaR)										
$\text{VaR}_{5\%}$	381.59	377.18	382.78	377.83	381.08	377.95	380.19	380.29	376.03	378.72
$\text{VaR}_{5\%}$ w/o insurance	316.28	325.91	316.28	325.91	316.28	325.91	316.28	325.91	316.28	325.91
$\text{VaR}_{5\%}$ improvement	65.31	51.27	66.50	51.92	64.80	52.04	63.92	54.38	59.75	52.81

M Protecting corn price risk

Previously we focus on discussing index insurance for production losses, i.e., yield insurance. However, as corn prices fluctuate, one might consider simultaneously providing corn price protection to farmers. In this section, we apply the same NN-based framework and design an index insurance contract to protect both the production and price risks, that is, the revenue protection.

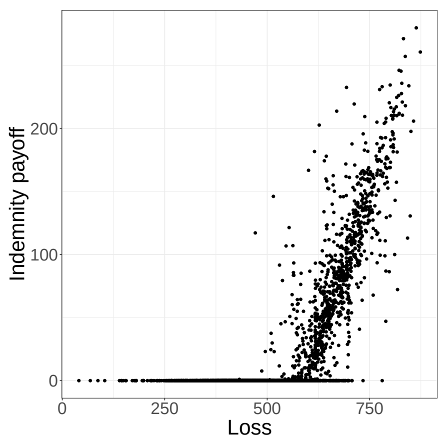
We use the average price of the Chicago Mercantile Exchange (CME) Group December futures contracts during the month of February as the expected corn price.[3] The futures price contains the market's expectation of the corn commodity demand and supply within the same calendar year. In addition, the average of December CME Group futures contract price during February is also used as the projected price of the revenue protection in the FCIP in the U.S. Therefore, it is an appropriate measure of price risk. The sample period for the futures prices is from 1980 to 2017. We compare two contracts: *NN72* (the baseline model), and *Linear72* (a linear contract with all 72 weather indices).

Table M.1 summarizes the index insurance performances. We see that after considering price risk, the *NN72* index insurance remains effective in improving farmers' utilities and CEW, stabilizing their wealth distributions, and reducing downside tail risks. The *NN72* contract achieves a CEW improvement of \$19.46/acre in the training sample and \$20.68/acre in the test sample, improving CEW by 4.85% and 5.19% in the training sample and test sample, respectively. This is similar to the case that considers production risk only in Table 4. Comparing *NN72* and *Linear72* contracts, we see that *Linear72* has much worse utility and CEW improvement, even though the premium is similar. Figure M.1 shows that *NN72* achieves a more effective basis risk reduction than *Linear72*.

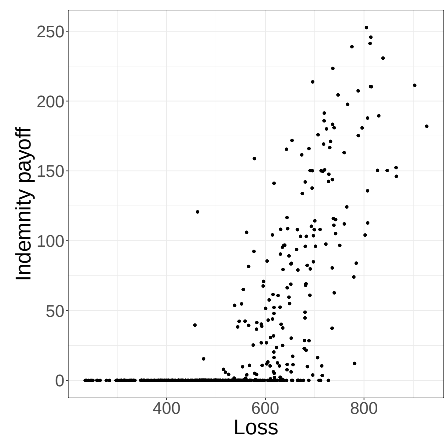
[3] See Rouwenhorst and Tang (2012) and Kang et al. (2020) for discussions of commodity pricing.

Table M.1: **Protecting both production and price risks.** We consider an index insurance contracts protecting both production and price risks. The *NN72* contract has the 3-hidden-layer (64-64-16 neurons) structure, as in the baseline model. The *Linear72* is a linear contract with all 72 weather indices. Panel A summarizes utilities with and without (w/o) different index insurance policies and the percentage of utility improvement. Panel B summarizes certainty equivalent wealth (CEW) with and without (w/o) index insurance policies and the CEW improvements in dollars and as a percentage. Panel C summarizes policy characteristics, including premiums, coverage, and profits for the insurer. Panel D summarizes the risk reduction effect of different index insurance policies, measured by the standard deviation of wealth. Panel E summarizes the risk reduction at the tail, measured by the 5%-level value-at-risk (VaR). The risk loading parameter at equilibrium (λ^*) for each contract is reported in parentheses. The sample period is 1980-2017.

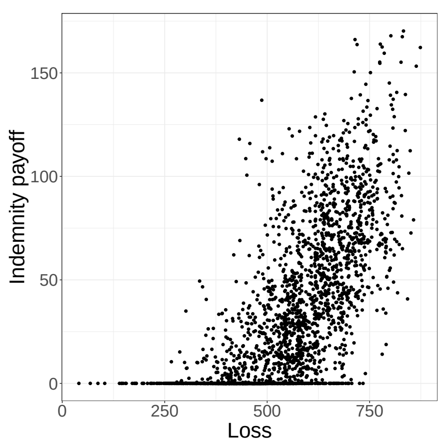
Contract	NN72 ($\lambda^* = 1.4137$)		Linear72 ($\lambda^* = 1.3791$)	
Data	Training	Test	Training	Test
Panel A: Utility improvement				
U with insurance	-4.32	-4.37	-4.68	-4.63
U w/o insurance	-5.05	-5.15	-5.05	-5.15
U improvement (%)	14.41%	15.25%	7.47%	10.11%
Panel B: CEW improvement				
CEW with insurance	420.50	419.32	410.76	411.95
CEW w/o insurance	401.05	398.63	401.05	398.63
CEW improvement	19.46	20.68	9.71	13.32
CEW improvement (%)	4.85%	5.19%	2.42%	3.34%
Panel C: Policy characteristics				
Premium	42.28	40.53	41.33	38.15
Coverage	29.91	28.67	29.97	27.66
Insurer Profit	12.37	11.86	11.36	10.49
Panel D: Risk reduction measured by standard deviation				
Std	100.67	102.83	110.20	112.46
Std w/o insurance	132.67	132.65	132.67	132.65
Std reduction	24.12%	22.48%	16.94%	15.22%
Panel E: Risk reduction measured by Value-at-Risk (VaR)				
$\text{VaR}_{5\%}$	333.10	299.72	289.48	288.87
$\text{VaR}_{5\%}$ w/o insurance	256.32	251.00	256.32	251.00
$\text{VaR}_{5\%}$ improvement	76.79	48.71	33.17	37.86



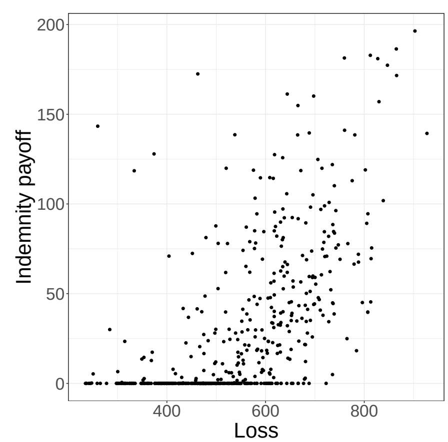
(a) NN72, training set



(b) NN72, test set



(c) Linear72, training set



(d) Linear72, test set

Figure M.1: **Basis risk of index insurance protecting both production and price risks.** These figures plot the insurance payoffs against actual loss, over the training or test set. The index insurance is designed to protect both production and price risks. The *NN72* contract has a 3-hidden-layer (64-64-16 neurons) structure, as in the baseline model. The *Linear72* is a linear contract with 72 weather indices.

N Regulatory costs

The NN-based insurance contract seems to be more complicated than traditional insurances and such contract complexity might increase litigation risks and regulatory frictions. In this section, we further evaluate the impacts of tightening regulatory costs. We quantify the regulatory costs of contract complexity by the regulatory capital reserve. The Solvency II directive has the solvency capital requirement (SCR) to achieve solvency with a 99.5% probability over a one-year horizon. SCR serves as a "soft" supervisory specification, and in practice the actual capital that an insurer has to hold is capped and floored at 50% and 20% of SCR (Towers Watson 2010). In this section, we test the impacts of supply-side frictions when an insurer reserves regulatory capital amounted to $\{20\%, 35\%, 50\%\}$ of the SCR. We assume the regulatory costs will be priced into the insurance premium, $\pi_e(I)$, as follows,

$$\pi_e(I) := \frac{\lambda}{n} \sum_{j=1}^{n} I(\boldsymbol{x}_j) + \text{Regulatory Capital Holding} \times \text{Cost of Capital.}$$

We assume the insurer's cost of capital is 7%, which is the industry's weighted average cost of capital, according to S&P Global Ratings.

Results are summarized in Table N.1. We see that the farmer's utility and CEW improvements decrease with the regulatory capital holding. But, the index insurance remains effective in utility improvement and basis risk reduction even with the presence of additional regulatory costs.

Table N.1: **Impacts of regulatory costs.** This table presents the impacts of regulatory costs, captured by the regulatory capital reserve of insurers. Columns 2-7 display results for different regulatory capital holding levels as a percentage of solvency capital requirement (20%, 35%, and 50%). The last two columns correspond to our baseline model without regulatory costs. Panel A summarizes utilities with and without (w/o) different index insurance policies and the percentage of utility improvement. Panel B summarizes CEW with and without (w/o) index insurance policies and certainty equivalent wealth (CEW) improvements in dollars and as a percentage. Panel C summarizes policy characteristics, including premiums, coverage, and profits for the insurers (net of regulatory cost). Panel D summarizes the risk reduction effect of different index insurance policies, measured by the standard deviation of wealth. Panel E summarizes the risk reduction at the tail, measured by the 5%-level value-at-risk (VaR).

Capital Reserving (as % of SCR)	20%		35%		50%		BL	
Data	Training	Test	Training	Test	Training	Test	Training	Test
Panel A: Utility improvement								
U with insurance	-3.58	-3.59	-3.59	-3.61	-3.60	-3.61	-3.57	-3.57
U w/o insurance	-3.99	-4.16	-3.99	-4.16	-3.99	-4.16	-3.99	-4.16
U improvement (%)	10.22%	13.85%	9.93%	13.38%	9.60%	13.17%	10.60%	14.35%
Panel B: CEW improvement								
CEW with insurance	444.10	443.90	443.70	443.22	443.25	442.91	444.64	444.61
CEW w/o insurance	430.63	425.26	430.63	425.26	430.63	425.26	430.63	425.26
CEW improvement	13.47	18.64	13.07	17.96	12.62	17.65	14.00	19.36
CEW improvement (%)	3.13%	4.38%	3.03%	4.22%	2.93%	4.15%	3.25%	4.55%
Panel C: Policy characteristics								
Premium	29.40	29.40	29.73	29.46	30.09	29.83	28.44	28.72
Coverage	23.21	23.22	23.13	22.92	23.08	22.86	22.91	23.13
Insurer Profit	5.60	5.61	5.58	5.53	5.57	5.52	5.53	5.59
Panel D: Risk reduction measured by standard deviation								
Std	53.77	47.71	53.75	48.39	53.82	48.18	54.05	47.49
Std w/o insurance	81.94	78.92	81.94	78.92	81.94	78.92	81.94	78.92
Std reduction	34.38%	39.55%	34.41%	38.68%	34.33%	38.95%	34.04%	39.82%
Panel E: Risk reduction measured by Value-at-Risk (VaR)								
$\text{VaR}_{5\%}$	382.94	376.51	382.39	375.72	381.93	374.85	382.89	379.64
$\text{VaR}_{5\%}$ w/o insurance	316.28	325.91	316.28	325.91	316.28	325.91	316.28	325.91
$\text{VaR}_{5\%}$ improvement	66.67	50.60	66.11	49.81	65.65	48.94	66.61	53.73

O Performance of other machine learning models

In this section, we consider alternative machine learning models that could capture high-dimensionality and nonlinearity. We focus on regression tree models and support vector machine (SVM), as our purpose is not to run an exhaustive comparison between NN and other machine learning methods. We consider different tree-based models, including a simple regression tree, tree bagging, random forest, and tree boosting (Rossi and Timmermann 2015, Rossi and Utkus 2021, Li and Rossi 2021, Cong et al. 2022).[4] Tree "bagging", short for bootstrap aggregation, involves bootstrapping many training samples, building a separate tree model using each training set, and averaging their predictions. Random forest improves over tree bagging by decorrelating the bagged trees. This is achieved by randomly drawing some predictors at each splitting branch. Tree boosting improves the performance of regression tree models by sequentially combining a series of simple, small trees. For detailed introduction of different tree-based models, see Hastie et al. (2009) and Gu et al. (2020).

Table O.1 presents the results. The utility improvements in the test set are 5.70%, 6.00%, 8.61%, 7.95% and 6.73% for single tree, tree bagging, random forest, tree boosting, and SVM, respectively. Comparing with the results in Table 4, we see:

1. Tree models and SVM perform better than low-dimensional linear models (the utility improvement is 0.55% for *Linear1* and 3.08% for *Linear5*) and polynomial models (the utility improvement is 3.28% for *Quad5* and 3.84% for *Cubic5*), due to their ability to capture high-dimensionality and non-linearity.

2. Among tree models, random forest and tree boosting perform best and their performances are similar to the high-dimensional linear model *Linear72* or the low-dimensional NN model *NN5*.

[4] Rossi and Timmermann (2015) use boosted regression trees to construct the covariance risk measure in an intertemporal CAPM setting. Rossi and Utkus (2021) use boosted regression trees to explain the cross-sectional heterogeneity in the effects of robo-advising on portfolio allocations and investment performance. Li and Rossi (2021) use boosted regression trees to predict mutual fund performances. Cong et al. (2022) consider the cross-sectional dependence structure among asset returns in tree-based models and highlight the importance of sequential sorting offered by tree-based models.

3. Overall, all tree models and SVM perform worse than *NN72*.

These results are in line with the literature. For example, Bianchi et al. (2021) mention that tree-based methods and NN are the best-performing methods. Gu et al. (2020) also find that trees and NN improve predictions but NN dominates tree-based methods.

The underperformance of tree-based models relative to NN models may be due to the fact that individual features might not be good predictors compared to linear combinations of features in our problem (Hastie et al. 2009). In addition, NN-based models yield smooth outputs which are desirable for designing insurance payoff functions. Therefore, we choose *NN72* as the baseline model. However, one must be cautious that the above comparison doesn't consider the model complexity and interpretability, which could make tree-based models more favourable.

Table O.1: **Alternative machine learning models.** We compare the performance of our baseline model ($NN72$) with alternative machine learning models, including tree-based models and SVM model. Panel A summarizes utilities with and without (w/o) different index insurance policies and the percentage of utility improvement. Panel B summarizes CEW with and without (w/o) index insurance policies and certainty equivalent wealth (CEW) improvements in dollars and as a percentage. Panel C summarizes policy characteristics, including premiums, coverage, and profits for the insurers. Panel D summarizes the risk reduction effect of different index insurance policies, measured by the standard deviation of wealth. Panel E summarizes the risk reduction at the tail, measured by the 5%-level value-at-risk (VaR). "BL" represents the baseline case studied in Section 4.2. The risk loading parameter at equilibrium (λ^*) for each contract is reported in parentheses.

Model	NN72 (BL) ($\lambda^* = 1.2414$)	Single Tree ($\lambda^* = 1.2126$)	Tree Bagging ($\lambda^* = 1.2772$)	Random Forest ($\lambda^* = 1.2766$)	Tree Boosting ($\lambda^* = 1.2554$)	SVM ($\lambda^* = 1.2637$)
Panel A: Utility improvement						
U with insurance	-3.57	-3.93	-3.91	-3.80	-3.83	-3.88
U w/o insurance	-4.16	-4.16	-4.16	-4.16	-4.16	-4.16
U improvement (%)	14.35%	5.70%	6.00%	8.61%	7.95%	6.73%
Panel B: CEW improvement						
CEW with insurance	444.61	432.59	432.99	436.51	435.61	433.97
CEW w/o insurance	425.26	425.26	425.26	425.26	425.26	425.26
CEW improvement	19.36	7.34	7.74	11.25	10.35	8.71
CEW improvement (%)	4.55%	1.72%	1.82%	2.65%	2.43%	2.05%
Panel C: Policy characteristics						
Premium	28.72	24.18	17.32	18.59	20.48	18.58
Coverage	23.13	19.94	13.56	14.56	16.31	14.70
Insurer Profit	5.59	4.24	3.76	4.03	4.17	3.88
Panel D: Risk reduction measured by standard deviation						
Std	47.49	69.66	67.66	63.78	64.62	66.10
Std w/o insurance	78.92	78.92	78.92	78.92	78.92	78.92
Std reduction	39.82%	11.73%	14.27%	19.18%	18.12%	16.25%
Panel E: Risk reduction measured by Value-at-Risk (VaR)						
$\text{VaR}_{5\%}$	379.64	333.58	337.40	339.78	337.53	339.06
$\text{VaR}_{5\%}$ w/o insurance	325.91	325.91	325.91	325.91	325.91	325.91
$\text{VaR}_{5\%}$ improvement	53.73	7.67	11.50	13.88	11.63	13.15

P Contract complexity measured by payout uncertainty

In Section 6.1.1, we assess the impact of contract complexity by considering the perceived value reduction effect in the index insurance payout. An alternative way to capture contract complexity is to add payout uncertainty. From the farmers' perspective, a highly complex contract that they do not understand effectively increases their perceived uncertainty about the insurance payout (Kubitza et al. 2020). More specifically, let $\epsilon \sim \mathrm{N}(0, \sigma_\epsilon^2)$ denote the experienced contract complexity, then conditional on their imperfect perception of the insurance contract, farmers' subjective beliefs about the indemnity payment is $\tilde{I}(\boldsymbol{X}) = I(\boldsymbol{X}) + \epsilon$, where $I(\boldsymbol{X})$ is the designed NN-based index insurance payout. Larger complexity aversion is represented by a larger value of σ_ϵ, indicating more difficulty for farmers to understand an insurance contract. We test different levels of payout uncertainty, i.e., $\sigma_\epsilon = 10$, 20, 30 and 40, which correspond to one half, 1, 1.5, and 2 times of expected insurance loss, respectively. Table P.1 summarizes the results.

Results in Table P.1 are largely consistent with Section 6.1.1. When complexity aversion increases, farmers' utility improvement from purchasing this insurance decreases. Nevertheless, even in the worst case when $\sigma_\epsilon = 40$, this NN-based insurance still performs similarly to *NN5* and *Linear72* and better than *Linear1*, *Linear5*, *Quadratic5*, and *Cubic5* in Table 4.

Table P.1: **Impacts of complexity aversion.** This table quantifies the impact of complexity aversion on insurance contracts. Contract complexity is captured by perceived payout uncertainty. Columns 4-11 present results for different levels of perceived payout uncertainty, i.e., $\sigma_\epsilon = 10$, 20, 30 and 40, which correspond to one half, 1, 1.5, and 2 times of expected insurance loss, respectively. Columns 2-3 correspond to our baseline model. Panel A summarizes utilities with and without (w/o) different index insurance policies and the percentage of utility improvement. Panel B summarizes CEW with and without (w/o) index insurance policies and certainty equivalent wealth (CEW) improvements in dollars and as a percentage. Panel C summarizes policy characteristics, including premiums, coverage, and profits for the insurers (net of regulatory cost). Panel D summarizes the risk reduction effect of different index insurance policies, measured by the standard deviation of wealth. Panel E summarizes the risk reduction at the tail, measured by the 5%-level value-at-risk (VaR).

	Baseline		$\sigma_\epsilon = 10$		$\sigma_\epsilon = 20$		$\sigma_\epsilon = 30$		$\sigma_\epsilon = 40$	
Data	Training	Test	Training	Test	Training	Test	Training	Test	Training	Test
Panel A: Utility improvement										
U with insurance	-3.57	-3.57	-3.58	-3.57	-3.61	-3.61	-3.66	-3.67	-3.74	-3.76
U w/o insurance	-3.99	-4.16	-3.99	-4.16	-3.99	-4.16	-3.99	-4.16	-3.99	-4.16
U improvement (%)	10.60%	14.35%	10.34%	14.16%	9.53%	13.31%	8.18%	11.83%	6.17%	9.77%
Panel B: CEW improvement										
CEW with insurance	444.64	444.61	444.27	444.34	443.15	443.11	441.30	440.99	438.60	438.11
CEW w/o insurance	430.63	425.26	430.63	425.26	430.63	425.26	430.63	425.26	430.63	425.26
CEW improvement	14.00	19.36	13.64	19.08	12.52	17.85	10.67	15.73	7.96	12.86
CEW improvement (%)	3.25%	4.55%	3.17%	4.49%	2.91%	4.20%	2.48%	3.70%	1.85%	3.02%
Panel C: Policy characteristics										
Premium	28.44	28.72	29.62	29.61	28.36	28.61	28.39	27.06	27.37	25.71
Coverage	22.91	23.13	23.86	23.85	22.85	23.05	22.87	21.79	22.05	20.71
Insurer Profit	5.53	5.59	5.76	5.76	5.52	5.56	5.52	5.26	5.32	5.00
Panel D: Risk reduction measured by standard deviation										
Std	54.05	47.49	54.40	47.68	57.65	51.17	61.69	56.67	67.50	63.17
Std w/o insurance	81.94	78.92	81.94	78.92	81.94	78.92	81.94	78.92	81.94	78.92
Std reduction	34.04%	39.82%	33.61%	39.58%	29.64%	35.16%	24.72%	28.19%	17.62%	19.96%
Panel E: Risk reduction measured by Value-at-Risk (VaR)										
$\text{VaR}_{5\%}$	382.89	379.64	379.25	376.91	370.75	368.77	361.70	356.76	350.36	346.74
$\text{VaR}_{5\%}$ w/o insurance	316.28	325.91	316.28	325.91	316.28	325.91	316.28	325.91	316.28	325.91
$\text{VaR}_{5\%}$ improvement	66.61	53.73	62.97	51.00	54.47	42.87	45.42	30.85	34.08	20.84

Q Out-of-state tests with a distant state

Our out-of-state tests in Section 5.1 show that NN-based insurance trained on Illinois data can work reasonably well in adjacent states. In this appendix, we perform an alternative distant out-of-state test. That is, we use North Dakota, a state in corn belt but geographically distant from Illinois, as a negative test sample. More specifically, we conduct the following two tests. First, similar to the adjacent state tests, we train an NN-based index insurance contract based on Illinois data and test its performance using the data from North Dakota. Second, we train and test an NN-based index insurance based on the data from North Dakota. Results of these two tests are presented in Table Q.1.

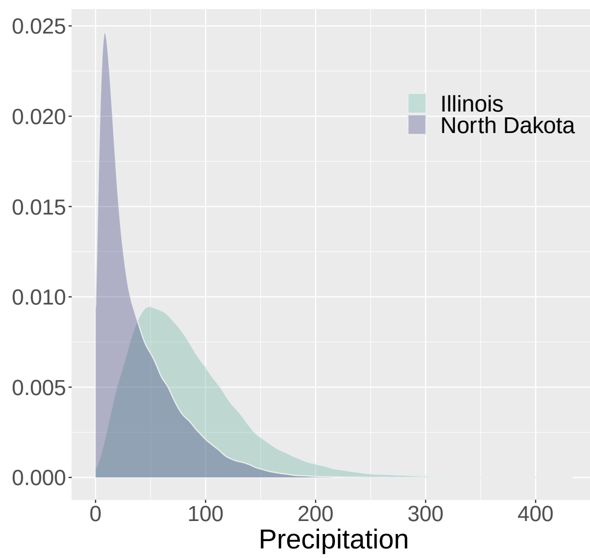
As expected, Columns (3) and (4) show that the North Dakota-trained model achieves performance similar to our baseline results in Illinois. It improves farmers' utility by 12.40% (11.04%) in the training (test) sample. This suggests the generality of our index insurance design framework. However, Columns (1) and (2) show that the Illinois-trained model does not perform well in the test sample of North Dakota. For example, Column (2) shows that farmers' utility with insurance is lower than the case without insurance. This is not surprising because the weather patterns and the interaction of different weather indices as well as nonlinear mapping of weather indices to production losses in these two states are very different, due to their distant geographical locations. The latitude and longitude of Illinois are 47.5515°N and 101.0020°W, respectively, while for North Dakota, they are 40.6331°N, 89.3985°W. Also, the distributions of weather indices exhibit significant differences between Illinois and North Dakota, as summarized in Table Q.2 and Figure Q.1. For example, a low precipitation event recognized as an indicator of a large loss by the Illinois-trained model, may not be considered as a low precipitation event in North Dakota because precipitations are generally much lower there.

Table Q.1: **Out-of-state tests, using a distant state.** We compare the performances of different NN-based insurance contracts. Column (1) uses Illinois data to train the model and then Column (2) tests the model in North Dakota. Column (3) uses North Dakota data to train the model and then Column (4) tests the model in North Dakota. Panel A summarizes utilities with and without (w/o) different index insurance policies and the percentage of utility improvement. Panel B summarizes CEW with and without (w/o) index insurance policies and certainty equivalent wealth (CEW) improvements in dollars and as a percentage. Panel C summarizes policy characteristics, including premiums, coverage, and profits for the insurers. Panel D summarizes the risk reduction effect of different index insurance policies, measured by the standard deviation of wealth. Panel E summarizes the risk reduction at the tail, measured by the 5%-level value-at-risk (VaR).

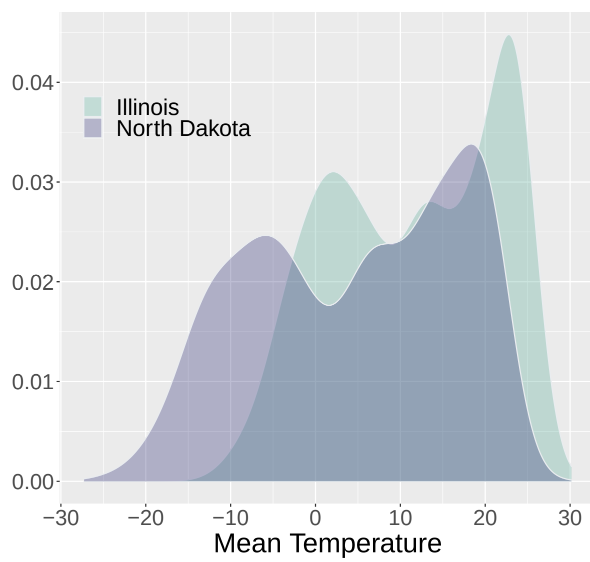
	(1) Training	(2) Test	(3) Training	(4) Test
Data	Illinois	North Dakoda	North Dakoda	North Dakoda
Panel A: Utility improvement				
U with insurance	-3.55	-3.86	-3.20	-3.23
U w/o insurance	-4.02	-3.63	-3.66	-3.63
U improvement (%)	11.63%	-6.41%	12.40%	11.04%
Panel B: CEW improvement				
CEW with insurance	445.21	434.58	457.98	456.97
CEW w/o insurance	429.75	442.35	441.43	442.35
CEW improvement	15.46	-7.76	16.55	14.62
CEW improvement (%)	3.60%	-1.76%	3.75%	3.31%
Panel C: Policy characteristics				
Premium	29.09	34.35	37.99	34.86
Coverage	23.43	27.67	31.17	28.60
Insurer Profit	5.66	6.68	6.82	6.26
Panel D: Risk reduction measured by standard deviation				
Std	51.29	107.29	84.72	75.15
Std w/o insurance	80.60	103.22	112.66	103.22
Std reduction	36.36%	-3.95%	24.80%	27.19%
Panel E: Risk reduction measured by Value-at-Risk (VaR)				
$\text{VaR}_{5\%}$	381.12	313.13	356.71	368.76
$\text{VaR}_{5\%}$ w/o insurance	317.70	306.56	287.07	306.56
$\text{VaR}_{5\%}$ improvement	63.42	6.57	69.64	62.20

Table Q.2: **Comparing weather conditions in Illinois and North Dakota.** This table summarizes annual weather conditions in Illinois and North Dakota, including mean, standard deviation (Std), median (Q_{50}), 25% quantile (Q_{25}), and 75% quantile (Q_{75}).

Panel A: Annual summary of weather for Illinois						
	pcpn	dpt	tmin	vpdmax	vpdmin	tmax
Mean	82.57	5.24	5.79	12.40	1.06	17.44
Std	50.51	9.12	9.35	7.88	0.65	10.59
Q_{25}	45.66	-2.52	-2.11	4.93	0.58	7.95
Q_{50}	72.87	5.08	6.12	12.05	0.91	18.91
Q_{75}	108.88	13.95	14.57	18.56	1.40	27.33
Panel B: Annual summary of weather for North Dakoda						
	pcpn	dpt	tmin	vpdmax	vpdmin	tmax
Mean	37.05	-1.99	-1.80	10.74	0.88	11.32
Std	35.13	10.32	11.40	8.71	0.74	13.06
Q_{25}	10.88	-10.17	-11.39	2.50	0.33	-0.50
Q_{50}	24.81	-1.78	-0.64	9.39	0.66	13.17
Q_{75}	53.59	7.53	8.54	16.97	1.23	23.16



(a) Precipitation



(b) Temperature

Figure Q.1: **Precipitation and temperature in Illinois and North Dakota.** These panels compare the distribution densities of precipitation (Panel (a)) and temperature (Panel (b)) in Illinois and North Dakota.

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