## Internet Appendix to

# "Time-to-Produce, Inventory, and Asset Prices" 

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#### Abstract

This appendix provides supplementary materials to the paper Time-to-Produce, Inventory, and Asset Prices (to appear at Journal of Financial Economics).


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## Appendices

## A. The projection procedures

This section describes the numerical computing strategy, the projection method, used in the calibrations. ${ }^{1}$

## A.1. Solving macroeconomic quantities

To minimize the computing burden, I first solve a bare-bones set of macroeconomic variables. The equilibrium conditions of this model can be described as

$$
\begin{equation*}
\mathbb{E}_{t}\left[\mathbf{F}\left(\mathbf{Y}_{t}, \mathbf{X}_{t}, \ldots, \mathbf{Y}_{t+d+1}, \mathbf{X}_{t+d+1}\right)\right]=0 \tag{A.1}
\end{equation*}
$$

where $\mathbf{X}$ and $\mathbf{Y}$ are the state variables and other variables of interest, respectively.
Extra care should be taken when handling the recursive preferences. To make it consistent with the general description in (A.1), we need to introduce one auxiliary variable to capture

[^1]the expected future utility, i.e., $e u_{t}=\mathbb{E}_{t} U_{t+1}^{1-\gamma}$. Summarizing, the equilibrium conditions are:
\[

$$
\begin{align*}
& \mathbb{E}_{t}\left[M_{t, t+1}\right]=-\mathbb{E}_{t}\left[M_{t, t+1} \alpha \omega Z_{t+1}^{1-\alpha}\left(K_{t-d+1}^{1-\omega} W_{t}^{\omega}\right)^{\alpha-1} K_{t-d+1}^{1-\omega} W_{t}^{\omega-1}\right]+\frac{\partial h_{t}}{\partial W_{t}}+1, \\
& 0=\mathbb{E}_{t}\left[M_{t, t+h} q_{t+h} \frac{\partial g_{t+h}}{\partial S_{t}}\right]-\mathbb{E}_{t}\left[\sum_{i=0}^{h} M_{t, t+i} w_{i}\right] \\
& q_{t}=\mathbb{E}_{t}\left[M_{t, t+1} q_{t+1}\right]-\delta \mathbb{E}_{t}\left[\sum_{i=1}^{d+1} M_{t, t+i} u_{i-1} q_{t+i}\right]+ \\
& \mathbb{E}_{t}\left\{M_{t, t+d+1}\left[\alpha(1-\omega) K_{t+1}^{-\omega} W_{t+d}^{\omega} Z_{t+d+1}^{1-\alpha}\left[K_{t+1}^{1-\omega} W_{t+d}^{\omega}\right]^{\alpha-1}+q_{t+d+1} \frac{\partial g_{t+d+1}}{\partial K_{t+1}}-\frac{\partial h_{t+d+1}}{\partial K_{t+1}}\right]\right\} \\
& C_{t}=Z_{t}^{1-\alpha}\left[K_{t-d}^{1-\omega} W_{t-1}^{\omega}\right]^{\alpha}-\sum_{i=0}^{h} w_{i} S_{t-i}-h_{t}-\left(W_{t}-W_{t-1}\right),  \tag{A.2}\\
& I_{t}=\sum_{i=0}^{h} w_{i} S_{t-i}, \\
& e u_{t}=\mathbb{E}_{t} U_{t+1}^{1-\gamma}, \\
& U_{t}=\left\{(1-\beta) C_{t}^{\frac{1-\gamma}{\theta}}+\beta\left[\mathbb{E}_{t} U_{t+1}^{1-\gamma}\right]^{\frac{1}{\theta}}\right\}^{\frac{\theta}{1-\gamma}}, \\
& K_{t+1}=K_{t}+g_{t}-\delta \sum_{i=0}^{d} K_{t-i} u_{i} .
\end{align*}
$$
\]

The pricing kernel is

$$
\begin{equation*}
M_{t, t+1}=\beta\left[\frac{C_{t+1}}{C_{t}}\right]^{-\frac{1}{\psi}}\left[\frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_{t} U_{t+1}^{1-\gamma}}\right]^{1-\frac{1}{\theta}} . \tag{A.3}
\end{equation*}
$$

The above equilibrium conditions consist of eight variables $\mathbf{Y}=(K, U, C, W, S, I, q$, $e u)$, and seven state variables $\mathbf{X}=\left(K_{t-2}, K_{t-1}, K_{t}, S_{t-2}, S_{t-1}, W_{t-1}, z_{t}\right)$.

I approximate the above 8 variables by the projection method. Given the number of state variables in the model, it is numerically challenging to compute the multidimensional integration for the conditional expectations and Galerkin weighted residuals. To avoid the curse of dimensionality, I apply non-product monomial rules instead of a full tensor grid, where the number of grids only grows quadratically with the number of state variables.

These rules are derived by Hammer and Stroud (1958) and Stroud and Secrest (1963) (see also Judd, 1998; Pichler, 2011).

The projection method can be outlined as follows:

- Step 1: Compute the deterministic steady state values of these variables and use these values to determine the approximation interval (the state space) for each state variable in the projection.

From the equilibrium conditions, the deterministic steady state can be solved from the following equations:

$$
\begin{align*}
\beta & =-\beta \alpha \omega\left(K^{1-\omega} W^{\omega}\right)^{\alpha-1} K^{1-\omega} W^{\omega-1}+\eta\left(\frac{W}{K}\right)^{\tau-1}+1  \tag{A.4}\\
q & =\sum_{i=0}^{h} \beta^{i} w_{i} / \beta^{h},  \tag{A.5}\\
q & =\frac{\beta^{d+1}\left[\alpha(1-\omega) K^{-\omega} W^{\omega}\left(K^{1-\omega} W^{\omega}\right)^{\alpha-1}-\frac{\eta}{\tau}(1-\tau)\left(\frac{W}{K}\right)^{\tau}\right]}{1-\beta+\delta \sum_{i=1}^{d+1} \beta^{i} u_{i-1}},  \tag{A.6}\\
C & =\left(K^{1-\omega} W^{\omega}\right)^{\alpha}-\delta K-\frac{\eta}{\tau}\left(\frac{W}{K}\right)^{\tau} K  \tag{A.7}\\
I & =\delta K  \tag{A.8}\\
e u & =U^{1-\gamma}  \tag{A.9}\\
U & =C  \tag{A.10}\\
S & =\delta K \tag{A.11}
\end{align*}
$$

where all variables without subscripts denote the steady state values and the steady state of the productivity shock is normalized to 1 .

- Step 2: Approximating variables of interest (Y) with a complete set of Chebyshev polynomials as

$$
\begin{equation*}
Y_{j}\left(X ; \kappa^{j}\right)=\sum_{i=1}^{n_{\kappa}} \kappa_{i}^{j} \phi_{i}(\varphi(X)), \tag{A.12}
\end{equation*}
$$

for $j=1, \ldots, 8$, where $n_{\kappa}$ is the number of coefficients, $\kappa^{j}$ is the coefficient vector of variable $j, \varphi$ is a linear mapping function from the state space $X$ to $[-1,1]^{7}$, and $\phi_{i}(\varphi(X)) \in \Phi$ with

$$
\begin{equation*}
\Phi=\left\{\prod_{l=1}^{7} T_{i_{l}}\left(\varphi\left(X_{l}\right)\right) \mid \sum_{l=1}^{7} i_{l} \leq 2,0 \leq i_{1}, \ldots, i_{7}\right\} . \tag{A.13}
\end{equation*}
$$

and $T$ is the univariate Chebyshev polynomials. The choice of intervals for the state variables is often crucial for the numerical accuracy of a projection method. I use a trial-and-error approach to select intervals which are small and yet cover all simulated distributions. The intervals for the capital stock, new project size, inventory, and shocks are $[0.7 K, 1.8 K],[0.5 S, 2 S],[0.3 W, 2 W]$, and $[\operatorname{Ln}(0.8), \operatorname{Ln}(1.2)]$, respectively.

- Step 3: Computing the conditional expectations of equilibrium conditions. I use the non-product monomial rule with a degree of 5 to compute the integration of conditional expectations. Specifically, I use the rule $\left[C_{n} d 5\right]$, as in Pichler (2011).
- Step 4: Computing Galerkin weighted residuals.

Theoretically, the true solution should satisfy the equilibrium conditions in (A.2) over the whole state space, i.e., the approximation residuals $\mathcal{R}(F)=0$. However, there are some residuals due to approximation errors. Galerkin method uses the basis functions of the Chebyshev polynomials as the weighted functions and compute the coefficients $\kappa$ by making the weighted residuals zero, as follows:

$$
\begin{equation*}
\mathcal{R}_{j, i}(\kappa)=\int_{X} \mathcal{R}_{j}(F(X ; \kappa)) \phi_{i}(\varphi(X)) d X=0 \tag{A.14}
\end{equation*}
$$

for $j=1, \ldots, 8$ and $i=1, \ldots, n_{\kappa}$.
I use the non-product monomial rule with a degree of 5 to compute the multidimensional integrations of Galerkin weighted residuals, as in Pichler (2011). Specifically, I use the rule $\left[E_{n}^{r^{2}} d 5\right]$. To save computing costs, note that the above equations can be
simplified as

$$
\begin{equation*}
\int_{X} \mathcal{R}_{j}(F(X ; \kappa)) \phi_{i}(\varphi(X)) d X=0=\int_{[-1,1]^{7}} \mathcal{R}_{j}\left(F\left(\varphi^{-1}(x) ; \kappa\right)\right) \phi_{i}(x) d x \tag{A.15}
\end{equation*}
$$

where $x=\varphi(X)$. So we can compute the integrals over $[-1,1]^{7}$ directly.

- Step 5: Finding the solution $\kappa$. The coefficient vector $\kappa$ is solved from the nonlinear equation system where the Galerkin weighted residuals are zero. I solve this system of nonlinear equations by a trust region approach provided in Intel Math Kernel Library. Another remaining issue is to choose the initial values for the solution. As suggested in Judd (1998), I compute the initial values from a third-order approximation of the model, as the following steps: (1) I first obtain a third-order approximation of the model. (2) Compute the simulated values for each variable from the third-order approximation, given the possible grid points of the state variables in the projection method. (3) Run the ordinary least square regressions of the simulated values against the complete set of Chebyshev polynomials and then use the estimated coefficients as the initial values.

The above procedures largely follow Pichler (2011), with several practical changes. First, the choice of intervals for state variables is often important for projection methods. Given the delays in production, it is possible that the state variables stay outside of the initial intervals during the iterations, which could lead to numerical instability and inaccuracy. Following Heer and Maußner (2008) I use larger intervals for approximating functions but integrate over smaller intervals when computing Galerkin weighted residuals so that the state variables always stay inside of the initial intervals during the iterations. Again, I use a trial-and-error approach to select smaller intervals when computing Galerkin weighted residuals to ensure that. Specifically, the smaller intervals chosen for the capital stock, new project size, inventory, and shocks are $[0.88 \mathrm{~K}, 1.55 \mathrm{~K}],[0.9 S, 1.4 S],[0.95 \mathrm{~W}, 1.45 \mathrm{~W}]$, and $[\operatorname{Ln}(0.9), \operatorname{Ln}(1.1)]$, respectively. Second, given the large number of coefficients to be solved
simultaneously, I use a trust region approach provided in Intel Math Kernel Library to solve the nonlinear equation system, because conventional solvers (like Newton solvers) often fail to find the roots. Third, to speed up numerical computing, all codes are written to utilize the vector/matrix computing capacities instead of loop structures.

## A.2. Solving asset prices

Based on the previous approximations of macroeconomic variables from the projection method, next I use the similar projection method to compute asset prices, as follows:
(1) Return on dividend claims

First, I use the projection method to approximate the stock price, $P_{t}$, from the following

$$
\begin{equation*}
P_{t}=\mathbb{E}_{t}\left[M_{t, t+1}\left(P_{t+1}+D_{t+1}\right)\right], \tag{A.16}
\end{equation*}
$$

where total payouts are computed as

$$
\begin{equation*}
D_{t}=\alpha Y_{t}-h_{t}-I_{t}-\left(W_{t}-W_{t-1}\right), \tag{A.17}
\end{equation*}
$$

the pricing kernel, capital stock, inventory, and total investment can be computed from the approximations in the previous subsection. The steady state value of stock price is

$$
\begin{equation*}
P=\frac{\beta}{1-\beta}\left[\alpha\left(K^{1-\omega} W^{\omega}\right)^{\alpha}-\delta K-\frac{\eta}{\tau}\left(\frac{W}{K}\right)^{\tau} K\right] . \tag{A.18}
\end{equation*}
$$

Then the return on dividend claims, $R_{D, t+1}$, can be computed as

$$
\begin{equation*}
R_{D, t+1}=\frac{P_{t+1}+D_{t+1}}{P_{t}} \tag{A.19}
\end{equation*}
$$

(2) Return on consumption claims

First, I use the projection method to approximate the logarithmic price of consumption
claims, $\operatorname{Ln}\left(P_{C, t}\right)$, from the following

$$
\begin{equation*}
P_{C, t}=\mathbb{E}_{t}\left[M_{t, t+1}\left(P_{C, t+1}+C_{t+1}\right)\right], \tag{A.20}
\end{equation*}
$$

where consumption and the pricing kernel can be computed from the approximations in the previous subsection. The steady state price of consumption claims is $P_{C}=\frac{\beta}{1-\beta} C$.

Then the return on consumption claims, $R_{C, t+1}$, can be computed as

$$
\begin{equation*}
R_{C, t+1}=\frac{P_{C, t+1}+C_{t+1}}{P_{C, t}} \tag{A.21}
\end{equation*}
$$

(3) The risk-free rate

I use the projection method to approximate the risk-free rate, $r_{f, t+1}$, from the following

$$
\begin{equation*}
1=\mathbb{E}_{t}\left[M_{t, t+}\left(1+r_{f, t+1}\right)\right], \tag{A.22}
\end{equation*}
$$

where the pricing kernel can be computed from the approximations in the previous subsection. The steady state value of the risk-free rate is $(1-\beta) / \beta$.
(4) Investment returns

From (29), define the marginal benefits of capital stock at time $t+1$ as $P_{K, t+1}$, then we can use the projection method to approximate the marginal benefits of productive capital from the following asset pricing equation

$$
\begin{equation*}
1=\mathbb{E}_{t}\left[M_{t, t+} \frac{P_{K, t+1}}{q_{t}}\right], \tag{A.23}
\end{equation*}
$$

where the pricing kernel and the marginal $q$ can be computed from the approximations in the previous subsection. The steady state value of the marginal benefits of capital stock equals $q / \beta$.

So, the return on the productive capital, $R_{K, t+1}$, is

$$
\begin{equation*}
R_{K, t+1}=\frac{P_{K, t+1}}{q_{t}} \tag{A.24}
\end{equation*}
$$

Similarly, from (30), define the marginal benefits of inventory investment at time $t+1$ as $P_{W, t+1}$, then we can use the projection method to approximate the marginal benefits of inventory investment from the following asset pricing equation

$$
\begin{equation*}
1=\mathbb{E}_{t}\left[M_{t, t+1} \frac{P_{W, t+1}}{\eta\left(\frac{W_{t}}{K_{t-2}}\right)^{\tau-1}+1}\right], \tag{A.25}
\end{equation*}
$$

where the pricing kernel and inventory can be computed from the approximations in the previous subsection. The steady state value of the marginal benefits of inventory investment is

$$
\begin{equation*}
P_{W}=\frac{1}{\beta}\left[1+\eta\left(\frac{W}{K}\right)^{\tau-1}\right] \tag{A.26}
\end{equation*}
$$

So, the return on the inventory investment, $R_{W, t+1}$, is

$$
\begin{equation*}
R_{W, t+1}=\frac{P_{W, t+1}}{\eta\left(\frac{W_{t}}{K_{t-2}}\right)^{\tau-1}+1} . \tag{A.27}
\end{equation*}
$$

Hence, the investment returns, $R_{I, t+1}$, can be defined as the value-weighted average returns on the productive capital and inventory investment if inventory is present:

$$
\begin{equation*}
R_{I, t+1}=\frac{R_{K, t+1} K_{t+1}+R_{W, t+1} W_{t}}{K_{t+1}+W_{t}} \tag{A.28}
\end{equation*}
$$

## B. Estimating TTB and TTP

In this section, I estimate the lengths of TTB and TTP from firm level data to guide the specification in the calibrations. The basic idea is to explore correlations between stock returns and the average $q$, future output, investment expenditures, and inventory to pin down the lengths of TTB and TTP.

## B.1. Empirical regression

From (24), the investment return, $R_{t+1}^{I}$, can be written in terms of marginal $q$ as follows:

$$
\begin{align*}
& R_{t+1}^{I}=\frac{\frac{\partial V_{t+1}}{\partial K_{t+1}}}{q_{t}}=\frac{1}{q_{t}}\left\{q_{t+1}-\delta \mathbb{E}_{t+1}\left[\sum_{i=1}^{d+1} M_{t+1, t+i} u_{i-1} q_{t+i}\right]+\right.  \tag{B.29}\\
& \left.\mathbb{E}_{t+1}\left[M_{t+1, t+d+1}\left(\alpha(1-\omega) K_{t+1}^{-\omega} W_{t+d}^{\omega} Z_{t+d+1}^{1-\alpha}\left(K_{t+1}^{1-\omega} W_{t+d}^{\omega}\right)^{\alpha-1}+q_{t+d+1} \frac{\partial g_{t+d+1}}{\partial K_{t+1}}-\frac{\partial h_{t+d+1}}{\partial K_{t+1}}\right)\right]\right\}
\end{align*}
$$

where $V_{t+1}$ is the firm value at time $t+1$. Thus investment return is related to $\left(q_{t+1}, \ldots, q_{t+d+1}\right)$, which implies that firm value $V_{t+1}$ is related to $q_{t+1}, \ldots, q_{t+d+1}$.

Since marginal $q$ is unobservable, we instead compute the average $q$ as

$$
\begin{equation*}
M B_{t}=\frac{P_{t}}{K_{t+1}}=q_{t}+\mathbb{E}_{t} M_{t, t+1}\left[\frac{V_{t+1}}{K_{t+1}}-\frac{\partial V_{t+1}}{\partial K_{t+1}}\right] . \tag{B.30}
\end{equation*}
$$

Hence, $M B_{t}$ can be represented by $\left(q_{t}, \ldots, q_{t+d}\right)$. That is, $q_{t+d}$ is associated with $M B_{t}$. So, firm value $V_{t+1}$, is related to $\left(M B_{t-d+1}, \ldots, M B_{t}, M B_{t+1}, \ldots, M B_{t+d+1}\right)$.

The stock return can be written as

$$
\begin{equation*}
R_{t+1}=\frac{V_{t+1}}{K_{t+1} M B_{t}} \tag{B.31}
\end{equation*}
$$

We see that the stock return $R_{t+1}$ is not only mechanically related to ( $M B_{t}, M B_{t+1}$ ) as in the standard production models, but also related to both historical and future average $q$, i.e., $\left(M B_{t-d+1}, \ldots, M B_{t}, M B_{t+1}, \ldots, M B_{t+d+1}\right)$. Therefore, the correlations between stock returns
and historical and future average $q$ tell us the length of TTP.
Additionally, we know that firm value $V_{t+1}$ should also include predetermined future net outputs up to $h+d+1$ periods, which result from unfinished production and partly installed projects because of TTB and TTP constraints. Also, there are continued investment expenditures up to $h$ periods for unfinished investment projects initiated in the previous periods, which should be excluded from the firm value. Last, inventories inherited from the previous period also contribute to firm value. Hence, the correlations between stock returns, average $q$ ratios, future net income, investment expenditures, and inventory give us a way to estimate the lengths of TTB and TTP. Summarizing, the empirical regression is ${ }^{2}$

$$
\begin{align*}
R_{t, t+1} & =\alpha+\sum_{i=1}^{d-1} \beta_{-i} M B_{t-i}+\beta_{0} M B_{t}+\sum_{i=1}^{d+1} \beta_{i} M B_{t+i}  \tag{B.32}\\
& +\beta_{\text {size }} \text { Log }\left(\text { Size }_{t}\right)+\beta_{W} \text { Inventory }_{t}+\sum_{i=1}^{h} \beta_{I, i} \text { Investment }_{t+i} \\
& +\sum_{i=1}^{h+d+1} \beta_{N I, i} \text { NetIncome }{ }_{t+i}+\beta_{I K} I_{t} / K_{t}+\varepsilon_{t+1}
\end{align*}
$$

where $R_{t, t+1}$ is the quarterly stock return from time $t$ to $t+1, M B$ is the ratio of market equity to book equity, Size $_{t}$ is the firm size, Inventory $y_{t}$ is the inventory at the end of time $t$ normalized by Size ${ }_{t}$, Investment is the investment expenditures for projects initiated in previous $h$ quarters normalized by Size $_{t}$, NetIncome is the cash flow in the future $h+d+1$ quarters normalized by Size $_{t}$, and $I / K$ is the ratio of investment to capital which has been found to be important in explaining asset returns in the literature. I employ the standard Fama and MacBeth (1973) cross-sectional regression to estimate the market wide delays.

## B.2. Data

Subject to data availability, the sample is from March 1984 to December 2012. Only common stocks (share codes 10 or 11) in the NYSE/Amex/Nasdaq are included. I exclude

[^2]all financial (SIC code between 6000 and 6999) and utility firms (SIC code between 4900 and 4999), and firms with fiscal quarters ending on a month other than March/June/September /December. The quarterly stock returns are computed from monthly stock returns obtained from CRSP. Financial data are obtained from the COMPUSTAT quarterly files. Specifically, $M B$ is computed as in Fama and French (1993). Size is the market capitalization at the end of a quarter. Inventories are from COMPUSTAT Item INVTQ. Investments are computed from COMPUSTAT Item CAPXY. ${ }^{3}$ Future cashflows are the net income available to common equities (Item IBCOMQ), and $I / K$ is the ratio of investment to total assets (Item ATQ). I assume a 3-month delay in financial reporting to match financial data with stock price data.

## B.3. Empirical results

To apply (B.32) as a regression, we need to assume the maximum production delays first. Since the above regression requires $h+2 d+1$ consecutive quarterly observations, to balance the number of sample firms and the estimation precision as well as given the four-quarter TTB assumption in Kydland and Prescott (1982), I assume market-wide delays are not more than five quarters. ${ }^{4}$

Table C. 1 reports the cross-sectional regression results. Starting with Panel A, which includes all sample firms, the stock returns are positively correlated with future market-tobook ratios (up to time $t+3$ ) while negatively related to historical market-to-book ratios (up to time $t-2$ ). Although the evidence from historical and future market-to-book ratios is not quite symmetric, Model (1) suggests $d=2$ or 3 . Model (2) adds size and inventory to the regressors. Not surprising, inventory is positively related to expected stock returns. Model (3) is a fully specified regression, including future net income and investment expenditures. The current stock returns are positively correlated with future net income up to time $t+5$. This suggests that $h+d=4$. In Model (4), the ratio of investment to capital is added to the

[^3]regression. Although there is a negative relation between the ratio of investment to capital and stock returns, the results for other regressors are largely the same.

To see the difference between manufacturing and non-manufacturing firms, I restrict the sample to manufacturing firms (with SIC code between 1000 and 4000) only in Panel B. Several interesting observations arise. First, the average $R^{2}$ shows that the model provides a better fit for manufacturing firms than that of the full sample in Panel A. Second, the delays in production appear to be longer than the full sample case in Panel A. For example, Model (3) estimates $d=2$ or 3 and $h+d=4$. Overall, the estimation in Panels A and B suggests that $d=2$ or 3 and $h=1$ or 2 . Therefore, I set $h=2$ and $d=2$ in the main model of calibrations.

## C. Numerical accuracy of perturbation methods: Policy and value functions

This section further compares policy functions and value function from the first-order, second-order, and third-order perturbations. Fig. D. 1 depicts the approximations of consumption, new projects, total investment, inventory, and utility over a capital interval of [ $80 \% \mathrm{~K}, 150 \% \mathrm{~K}]$, which covers almost the entire simulated sample distribution. All other state variables are set to their steady state values, except the capital stock $K_{t}$. The results indicate that the first-order and third-order approximations are quite close, while the second-order approximation is different. For example, in Panel (a), the first-order and third-order approximations show that consumption increases with capital stock; however, the second order approximation shows that consumption decreases with capital stock. The second-order approximation also diverts from the first-order and third-order approximations for inventory in Panel (d). This point is also clear from value function in Panel (e). Utility in Panel (e) shows that the second order approximation gives counterfactual results, i.e., utility decreases with capital stock. The discrepancy between the second order and the third
order approximation widens as capital stock increases. Such a counterfactual approximation of the utility function could cause large approximation errors in asset prices. This signals possible numerical inaccuracy of the second-order perturbation method in this model.

## D. Impulse responses

To understand the mechanism of the main model, Fig. E. 1 displays the impulse responses of main quantities, e.g., capital $(K)$, inventory $(W)$, output $(Y)$, new projects $(S)$, total investment $(I)$, and consumption $(C)$, after a positive, one-standard-deviation technology shock at time 1, as a percentage deviation from their steady state values. For comparison, the plots include the main model $(h=2, d=2)$, Benchmark 1 (no inventory, $h=0, d=0$ ), Benchmark $4(h=2, d=0)$, and Benchmark $6(h=0, d=2)$.

One striking feature in Panels (a)-(e) of Fig. E. 1 is that the TTB and TTP constraints generate cyclical patterns in macroeconomic quantities, but in different dimensions. As the TTB constraint slows the response of capital investment to the technology shock, we observe a two-period lag in the response of capital stock in Panel (a) for models with TTB $(h=2)$. Meanwhile, as inventory is more responsive to the shock than capital investment when TTB is present, inventory increases more significantly in the first two periods for models with TTB in Panel (b). This translates into the increase of output in the first three periods for models with TTB in Panel (c). Models with a TTB constraint reach the maximum output at time 3. The TTP constraint directly affects output, as shown in Panel (c). With TTP $(d=2)$ only, Benchmark 6 shows that output decreases at time 2 and 3 as the shock decays, but it increases after time 3 as the capital stock increases from time 1. The main model also displays an increasing output after time 5. As a result, we see that the size of new projects for models with a TTP constraint actually increases after time 3 in Panel (d). Panel (e) shows that total investment reaches the maximum at time 3 due to TTB. Similar to the new project size, total investment also increases after time 5 in models with a TTP constraint,
while it decreases in models without a TTP constraint. Again, Panel (e) shows that TTP makes total investment less volatile, as shown in Table 2.

## Table C. 1

## Estimating time-to-build and time-to-produce: Fama-MacBeth regressions

This table estimates time-to-build ( $h$ ) and time-to-produce ( $d$ ), from the following cross-sectional regression:

$$
\begin{aligned}
R_{t, t+1}= & \alpha+\sum_{i=1}^{d-1} \beta_{-i} M B_{t-i}+\beta_{0} M B_{t}+\sum_{i=1}^{d+1} \beta_{i} M B_{t+i}+\beta_{s i z e} \log \left(\text { Size }_{t}\right)+\beta_{W} \text { Inventory }_{t} \\
& +\sum_{i=1}^{h} \beta_{I, i} \text { Investment }_{t+i}+\sum_{i=1}^{h+d+1} \beta_{N I, i} \text { NetIncome } e_{t+i}+\beta_{I K} I_{t} / K_{t}+\varepsilon_{t+1}
\end{aligned}
$$

where $R_{t, t+1}$ is the quarterly stock return from time $t$ to $t+1, M B$ is computed as in Fama and French (1993), Size $_{t}$ is the market capitalization, Inventory ${ }_{t}$ is from Compustat Item INVTQ normalized by Size $e_{t}$, Investment is computed from Compustat Item CAPXY normalized by Size ${ }_{t}$, NetIncome is the net income available to common equities (Item IBCOMQ) normalized by Size $_{t}$, and $I / K$ is the ratio of investment to total assets (Item ATQ). Only common stocks (share codes 10 or 11) in the NYSE/Amex/Nasdaq are included, excluding all financial (SIC code between 6000 and 6999) and utility firms (SIC code between 4900 and 4999) and firms with fiscal quarters ending on a month other than March/June/September/December. All coefficients are in percentage and $t$ statistics are reported in parentheses. The sample period is $3 / 1984-12 / 2012$.

| Log $\left(\right.$ Size $\left._{t}\right)$ | (1) | (2) | (3) | (4) | (1) | (2) | (3) | (4) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Panel A: All individual stocks |  |  |  | Panel B: Manufacture firms |  |  |  |
|  |  | -0.78 | -1.04 | -1.03 |  | -0.70 | -1.00 | -0.98 |
|  |  | (-4.53) | (-6.73) | (-6.69) |  | (-4.07) | (-6.51) | (-6.48) |
| $M B_{t-3}$ | -0.10 | -0.09 | -0.07 | -0.06 | -0.18 | -0.17 | -0.14 | -0.13 |
|  | (-2.27) | (-2.13) | (-1.78) | (-1.71) | (-3.13) | (-3.02) | (-2.76) | (-2.66) |
| $M B_{t-2}$ | -0.33 | -0.29 | -0.28 | -0.28 | -0.33 | -0.30 | -0.28 | -0.28 |
|  | (-4.25) | (-4.38) | (-4.36) | (-4.33) | (-3.39) | (-3.35) | (-3.18) | (-3.21) |
| $M B_{t-1}$ | -0.30 | -0.28 | -0.28 | -0.27 | -0.41 | -0.39 | -0.38 | -0.38 |
|  | (-4.40) | (-4.23) | (-4.37) | (-4.33) | (-4.30) | (-4.30) | (-4.35) | (-4.39) |
| $M B_{t}$ | -1.95 | -1.89 | -1.87 | -1.87 | -2.36 | -2.29 | -2.27 | -2.26 |
|  | (-9.57) | (-9.54) | (-9.44) | (-9.46) | (-10.08) | (-10.01) | (-10.12) | (-10.13) |
| $M B_{t+1}$ | 2.50 | 2.49 | 2.48 | 2.47 | 3.09 | 3.07 | 3.06 | 3.05 |
|  | (10.61) | (10.47) | (10.50) | (10.57) | (10.28) | (10.22) | (10.20) | (10.26) |
| $M B_{t+2}$ | 0.62 | 0.63 | 0.61 | 0.62 | 0.55 | 0.56 | 0.55 | 0.55 |
|  | (5.56) | (5.58) | (5.63) | (5.65) | (4.60) | (4.63) | (4.74) | (4.72) |
| $M B_{t+3}$ | 0.20 | 0.20 | 0.22 | 0.22 | 0.24 | 0.24 | 0.24 | 0.24 |
|  | (2.54) | (2.48) | (2.74) | (2.79) | (2.34) | (2.18) | (2.23) | (2.26) |
| $M B_{t+4}$ | 0.04 | 0.07 | 0.11 | 0.11 | 0.07 | 0.11 | 0.16 | 0.15 |
|  | (0.70) | (1.30) | (2.14) | (2.09) | (0.94) | (1.58) | (2.17) | (2.16) |
| $M B_{t+5}$ | -0.02 | -0.03 | -0.00 | 0.00 | -0.01 | -0.01 | 0.02 | 0.02 |
|  | (-0.74) | (-0.91) | (-0.03) | (0.01) | (-0.26) | (-0.29) | (0.43) | (0.44) |
| Inventoryt |  | $1.31$ |  |  |  | 1.27 |  | 1.63 |
|  |  | (5.21) | (6.33) | (6.03) |  | (4.43) | (5.70) | (5.35) |
| NetIncome ${ }_{\text {t+1 }}$ |  |  | 17.89 | 17.94 |  |  | 19.66 | 19.72 |
|  |  |  | (7.92) | (7.96) |  |  | (7.92) | (7.97) |
| NetIncome ${ }_{t+2}$ |  |  | 22.27 | 22.32 |  |  | 24.13 | 24.18 |
|  |  |  | (10.00) | (10.05) |  |  | (9.21) | (9.26) |
| NetIncome ${ }_{t+3}$ |  |  | 11.55 | 11.52 |  |  | 13.06 | 12.96 |
|  |  |  | (4.69) | (4.68) |  |  | (4.68) | (4.66) |
| NetIncome ${ }_{t+4}$ |  |  | 10.10 | 10.06 |  |  | 12.26 | 12.23 |
|  |  |  | (5.73) | (5.72) |  |  | (5.33) | (5.36) |
| NetIncome ${ }_{t+5}$ |  |  | $8.14$ |  |  |  | $7.66$ | $7.55$ |
|  |  |  | $(4.74)$ | (4.69) |  |  | (4.01) | (3.97) |
| NetIncome ${ }_{t+6}$ |  |  | 2.63 | 2.67 |  |  | 4.17 | 4.26 |
|  |  |  | (1.50) | (1.54) |  |  | (2.26) | (2.33) |
| NetIncome ${ }_{t+7}$ |  |  | $0.48$ |  |  |  | $-1.28$ | -1.19 |
|  |  |  | $(0.35)$ | (0.37) |  |  | $(-0.70)$ | (-0.65) |
| NetIncome ${ }_{\text {+ }}$ |  |  | -0.23 | -0.13 |  |  | 0.13 | 0.15 |
|  |  |  | (-0.15) | (-0.08) |  |  | (0.06) | (0.07) |
| NetIncome ${ }_{\text {t+9 }}$ |  |  | -0.68 | -0.76 |  |  | 1.32 | 1.25 |
|  |  |  | (-0.39) | (-0.44) |  |  | (0.56) | (0.52) |
| Investment $_{t+1}$ |  |  | -9.11 | -7.80 |  |  | -22.08 | -20.65 |
|  |  |  | (-4.31) | (-3.71) |  |  | (-5.19) | (-4.41) |
| Investment $_{t+2}$ |  |  | 4.57 | 5.37 |  |  | 10.81 | 12.04 |
|  |  |  | (1.57) | (1.87) |  |  | (2.04) | (2.37) |
| Investment $_{t+3}$ |  |  |  |  |  |  |  |  |
|  |  |  | $(6.01)$ | (5.99) |  |  | $(4.64)$ | $(4.81)$ |
| Investment $_{t+4}$ |  |  | 25.62 | 26.55 |  |  | 45.43 | 47.33 |
|  |  |  | (9.77) | (10.07) |  |  | (6.46) | (6.97) |
| $I_{t} / K_{t}$ |  |  |  | -17.54 |  |  |  | -19.66 |
|  |  |  |  | (-3.20) |  |  |  | (-3.16) |
| $R^{2}$ | 0.107 | 0.125 | 0.172 | 0.174 | 0.126 | 0.145 | 0.206 | 0.208 |



Fig. D.1. Policy functions and value function
The policy and value functions with respect to the capital stock $K_{t}$ are plotted, using the first-order, second-order, and third-order perturbations. All other state variables are set to their steady state values.


Fig. E.1. Impulse response functions
This figure depicts the impulse response functions of various models after a positive, one-standard-deviation technology shock at time 1 . These plots include the standard RBC without inventory ( $h=0, d=0$ ), the standard RBC model with inventory and a 3-quarter time-to-build $(h=2, d=0)$, the standard RBC model with inventory and a 3-quarter time-to-produce ( $h=0, d=2$ ), and the standard RBC model with inventory, a 3-quarter time-to-build constraint, and a 3-quarter time-to-produce constraint $(h=2, d=2)$. Each variable is plotted as a percentage deviation from its steady state value.

## References

Hammer, P. C., Stroud, A. H., 1958. Numerical evaluation of multiple integrals II. Mathematics of Computation 12, 272-280.

Heer, B., Maußner, A., 2008. Computation of business cycle models: A comparison of numerical methods. Macroeconomic Dynamics 12, 641-663.

Judd, K. L., 1998. Numerical Methods in Economics. MIT Press, Cambridge, MA.

Kydland, F. E., Prescott, E. C., 1982. Time to build and aggregate fluctuations. Econometrica 50, 1345-1370.

Pichler, P., 2011. Solving the multi-country real business cycle model using a monomial rule Galerkin method. Journal of Economic Dynamics and Control 35, 240-251.

Stroud, A. H., Secrest, D., 1963. Approximate integration formulas for certain spherically symmetric regions. Mathematics of Computation 17, 105-135.


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[^1]:    ${ }^{1}$ I largely follow Pichler (2011), with several practical changes. Readers are strongly recommended to consult Pichler (2011) for technical details therein.

[^2]:    ${ }^{2}$ This empirical regression can be rigorously derived from a partial equilibrium producers' problem.

[^3]:    ${ }^{3}$ Since the detail investment plans are unavailable, here I use the total investment as a proxy for the investment expenditures of unfinished projects.
    ${ }^{4} \mathrm{I}$ also experimented with different maximum values of $h$ and $d$, and the results are similar.

