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Time-to-produce, inventory, and asset prices[☆]

Zhanhui Chen[*]

Division of Banking & Finance, Nanyang Business School, Nanyang Technological University, 50 Nanyang Avenue S3-B1B-72, 639798 Singapore

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Time-to-build, time-to-produce, and inventory have important implications for asset prices and quantity dynamics in a general equilibrium model with recursive preferences. Time-to-build captures the delay in transforming new investments into productive capital, and time-to-produce captures the delay in transforming productive capital into output. Both delays increase risks in that time-to-build generates procyclical payouts, whereas the time-to-produce amplifies this procyclicality. Inventory smooths consumption and helps capture interest rate volatility even when the elasticity of intertemporal substitution is small. The model is consistent with a high equity premium, a high stock return volatility, and lead-lag relations between asset prices and macroeconomic quantities.

1. Introduction

Recent production-based general equilibrium models have made significant progress towards understanding both asset prices and quantity dynamics. However, several challenges remain. First, payouts are counterfactually countercyclical and contribute little to the equity premium (Kaltenbrunner and Lochstoer, 2010). Second, when the elasticity of intertemporal substitution (EIS, hereafter) is small, the risk-free rate is excessively volatile and the term premium is abnormally large (Boldrin, Christiano, and Fisher, 2001; Jermann, 1998; Kaltenbrunner and Lochstoer, 2010). Third, the asset pricing role of inventories is largely overlooked, given its impact on the cost of capital (Belo and Lin, 2012; Jones and Tuzel, 2013). Fourth, the time-series interaction between asset prices and macroeconomic quantities has received little attention. For example, asset prices tend to lead quantities (Backus, Routledge, and Zin, 2007, 2010; Liu, Whited, and Zhang, 2009), a challenge to a standard real business cycle (RBC, hereafter) model in which everything moves simultaneously. This paper attempts to address these issues via production delay risks.

In their seminal time-to-build (TTB, hereafter) work, Kydland and Prescott (1982) consider a technology imperfection in building productive capital and define TTB as the delay in transforming new investment into productive

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* Tel.: +65 6790 6133; fax: +65 6791 3236.
E-mail address: chenzh@ntu.edu.sg

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capital. This paper extends Kydland and Prescott (1982) by incorporating another technology imperfection, namely, the delay in transforming productive capital into final goods, which I refer to as time-to-produce (TTP, hereafter). TTB and TTP are two natural yet different technology imperfections in the real world. First, the TTB constraint focuses on frictions during the formation of productive capital, while the TTP constraint focuses on frictions during the use of productive capital. Thus, current capital stock depends on investment projects initiated several periods ago under the TTB constraint, and current output depends on capital stock in place several periods ago under the TTP constraint. Second, the productivity of current capital stock is unobservable under the TTP constraint since the output of current capital is not realized until several periods later, while it is measurable under the TTB constraint.

These two production delays accumulate uncertainty and increase risks in the economy. Their impacts can be seen on the macroeconomic side. First, TTB slows the response of capital investment to productivity shocks, making it more difficult for agents to use capital investment to smooth consumption. In particular, when most investment expenditures occur in the later periods, TTB makes consumption extremely volatile because investment becomes less procyclical. But the good news is that TTB helps generate procyclical payouts. Inventories are necessary to smooth consumption under TTB. However, inventory holdings are too small under TTB, so consumption remains too volatile, compared with the case without TTB. Second, since the productivity of current capital stock is unobservable under TTP, capital investments cannot effectively smooth out the consumption and output volatilities caused by TTP. Thus, firms need to use inventory technology to smooth consumption under TTP. Only the TTP constraint ensures substantial inventory holdings observed in the data. Moreover, TTP amplifies the procyclical payouts, because capital investment becomes much riskier and less procyclical under the TTP constraint. Turning to the asset prices, the procyclical payouts lead to a high stock return volatility and a sizable equity premium. Additionally, as inventories are less risky and more responsive to the productivity shocks than the capital investment, inventories help generate a low volatility risk-free rate and a reasonable term premium even when EIS is small.

Given the number of state variables in this economy, I solve this model by a projection method with non-product monomial rules instead of a full tensor grid. The static and dynamic Euler equation errors show that the projection method is highly accurate and much more accurate than the first-order, second-order, and third-order perturbation methods. For example, the static and dynamic consumption errors from the projection method are at least an order of 2 smaller than those from the perturbation methods. Most dynamic consumption errors from the perturbation methods are as large as 6–10%, which cautions the application of perturbation methods in asset pricing models. Quantitatively, the main model reasonably matches both macroeconomic quantities and asset prices with the data. For example, the model generates a mean stock return of 5.28% and a volatility of 10.11% per year, compared with those of 5.53% and 12.03% for the unlevered returns in the data, respectively. The model also features a low risk-free rate volatility of 2.42% and a moderate term premium of 1.95%, together with an equity premium of 3.85% per year. The model exhibits the return predictability observed in the data as well (see Cochrane, 2008).

In this economy, current capital stock alone is not a sufficient statistic since inventories, TTB, and TTP expand the state space. Thus, stock returns and investment returns are usually different. Calibrations show that investment returns account for 79% of stock returns while contributing 93% to the volatilities. Expanded state space implies that asset prices contain more information than a single macroeconomic quantity. This explains the lead-lag patterns between asset prices and macroeconomic quantities documented by Backus, Routledge, and Zin (2007, 2010), the negative contemporaneous correlation between stock returns and investment growth (Liu, Whited, and Zhang, 2009), and the lagged investment effect in the investment regression (Eberly, Rebelo, and Vincent, 2012).

This paper builds on the large literature of production-based asset pricing models (e.g., Jermann, 1998; Boldrin, Christiano, and Fisher, 2001; Gomes, Kogan, and Zhang, 2003; Zhang, 2005; Kaltenbrunner and Lochstoer, 2010; Croce, 2014). These models introduce risks into the economy through investment frictions (e.g., convex capital adjustment costs, investment irreversibility, and capital immobility) or stochastic productivity shocks. In contrast, this paper emphasizes production delays. Only a few papers study the asset pricing implications of TTB. For example, Boldrin, Christiano, and Fisher (2001) investigate TTB under habit formation. This paper adds to the literature by studying TTP in a recursive preferences setting.

My paper also contributes to the business cycle literature. This paper motivates inventory from a consumption smoothing perspective and explores its asset pricing implications in a general equilibrium setting. This specification allows to explore the connection between inventories and the risk-free rate. In contrast, Belo and Lin (2012) and Jones and Tuzel (2013) examine the relation between inventory investment and stock returns in a partial equilibrium setting. Moreover, this paper constructs a general equilibrium model with production delays to endogenize the lead-lag patterns between asset prices and macroeconomic quantities. In contrast, Backus, Routledge, and Zin (2010) build a long-run risk model, assuming a positive correlation between consumption growth and stochastic volatility to capture such patterns.

The paper proceeds as follows. I first construct a production-based general equilibrium model and describe the numerical solution in Section 2. Section 3 outlines the data and parameters used in the calibrations. It also verifies the numerical accuracy of the projection method used. Section 4 presents the main numerical results. Finally, Section 5 concludes.

2. A general equilibrium model

Consider an all-equity representative firm that produces one real good and operates in a discrete and infinite time horizon. This assumption is abstract from the complications of real-world production, which features different goods and multiple levels of intermediate goods production.[1]

2.1. Firms

Firms use productive capital and inventories as input factors into the production (see, e.g., Kydland and Prescott, 1982; Gomes, Kogan, and Yogo, 2009; Belo and Lin, 2012; Jones and Tuzel, 2013). Time-to-produce is characterized by a delay equation,

$$Y_t = Z_t^{1-\alpha}\left[K_{t-d}^{1-\omega}W_{t-1}^{\omega}\right]^{\alpha}, \tag{1}$$

where Y_t is output at time t, Z_t is an aggregate productivity shock at time t, K_{t-d} is the capital stock at the beginning of time $t-d$, W_{t-1} is the inventories at the end of time $t-1$, α is the elasticity of capital, and ω is the elasticity of inventories.[2] Here, d denotes the time delay in production, capturing a TTP constraint (a delay of $d+1$ periods). That is, the output at time t depends on the production started at time $t-d$. Thus, the outputs in the next d periods are uncontrollable and predetermined by the historical capital stock levels. By setting $d=0$, a conventional production model is recovered.

To focus on business fluctuations, I assume that $z_t = \log Z_t$ follows a first-order autoregressive process without a trend,

$$z_{t+1} = \rho z_t + \sigma\, \varepsilon_{t+1}, \tag{2}$$

where $0 < \rho < 1$, ε_{t+1} denotes a standard normal distributed shock, and σ scales the shock. From (1) we see that the productivity of current capital stock (K_t) is unobservable since the output depends on Z_{t+d} and will be realized d periods later. When ρ is close to one, the conditional volatility of z_{t+d} is about $d\sigma^2$. Therefore, the TTP constraint accumulates uncertainty over the next d periods.

The firm problem also incorporates a TTB constraint, which impacts the capital stock evolution. Following (Kydland and Prescott, 1982), I assume there is a delay of $h+1$ periods in building productive capital. Let the motion of capital stock be

$$K_{t+1} = K_t + g_t - \delta\sum_{i=0}^{d} K_{t-i}u_i, \tag{3}$$

and

$$g_t = g(S_{t-h}, K_{t-d}), \tag{4}$$

where g_t is the capital installation function, S_{t-h} is the project size initiated at time $t-h$, and δ is the depreciation rate. The productive capital is assumed to be depreciated through $d+1$ periods with weight u_i at period i where $\sum_{i=0}^{d} u_i = 1$. We obtain the standard firm problem when $h=0$ and $d=0$, and the case of Kydland and Prescott (1982) when $h=3$ and $d=0$. We see that both TTB and TTP constraints increase the investment risks.

The capital installation function, g_t, is specified as in Jermann (1998), i.e.,

$$g_t = \left[\frac{a_1}{1-1/\chi}\left(\frac{S_{t-h}}{K_{t-d}}\right)^{1-1/\chi} + a_2\right]K_{t-d}, \tag{5}$$

where χ governs the capital adjustment costs, and a_1 and a_2 are constants. The capital adjustment costs are high when χ is low, and there are no capital adjustment costs when $\chi \to \infty$. As in Boldrin, Christiano, and Fisher (2001), the constants a_1 and a_2 are chosen so that there are no capital adjustment costs in the deterministic steady state,

$$a_1 = \delta^{1/\chi}, \quad a_2 = \frac{1}{1-\chi}\delta. \tag{6}$$

The total capital investment at time t, I_t, is

$$I_t = \sum_{i=0}^{h} w_i S_{t-i}, \tag{7}$$

where w_i is the investment expenditure weight of the project initiated at time $t-i$ with $\sum_{i=0}^{h} w_i = 1$.

In this model, I narrowly interpret inventories as the inventories of final goods, without referring to raw materials and work-in-process. Although there is no depreciation in inventories given the final good assumption, the firm pays storage costs and also faces inventory risk. For tractability, the inventory holding cost is specified similarly to the capital adjustment cost function. At time t, the inventory holding cost h_t is

$$h_t = h(W_t, K_{t-d}) = \frac{\eta}{\tau}\left(\frac{W_t}{K_{t-d}}\right)^{\tau} K_{t-d}, \tag{8}$$

where τ is the curvature parameter and η is the coefficient of inventory cost. Inventory holding cost h_t is captured as a proportion of productive capital since output depends on productive capital stock, and it is homogeneous of degree one in inventories and the productive capital stock.

2.2. Households

Agents are endowed with recursive preferences as follows:

$$U_t = \left\{(1-\beta)C_t^{\frac{1-\gamma}{\theta}} + \beta[\mathbb{E}_t U_{t+1}^{1-\gamma}]^{\frac{1}{\theta}}\right\}^{\frac{\theta}{1-\gamma}}, \tag{9}$$

where C_t is the consumption at time t, β is the time discount, γ measures the relative risk aversion, $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$, and ψ is the elasticity of intertemporal substitution. Then the pricing kernel is

$$M_{t,t+1} = \beta\left[\frac{C_{t+1}}{C_t}\right]^{-\frac{1}{\psi}}\left[\frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_t U_{t+1}^{1-\gamma}}\right]^{1-\frac{1}{\theta}}. \tag{10}$$

The representative agent owns and runs the firms. Total payouts are

$$D_t = \alpha Y_t - h_t - I_t - (W_t - W_{t-1}). \tag{11}$$

[1] It is necessary to define the boundary of firms and the input-output structure of the economy to incorporate intermediate goods production, which is beyond the scope of this paper.

[2] This specification assumes a unitary elasticity of substitution between capital stock and inventories, as in Gomes, Kogan, and Yogo (2009). This assumption could be relaxed. For example, Kydland and Prescott (1982) suggest a small elasticity of substitution of 0.2, while Belo and Lin (2012) set it as 2/3, and Jones and Tuzel (2013) set it from 1/3 to 1.

Therefore, the return on dividend claims, $R_{D,t+1}$, is

$$R_{D,t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}, \tag{12}$$

where P_t is the price of dividend claims at time t. As the model is calibrated to fit the aggregate consumption data, I also compute the aggregate consumption as

$$C_t = Y_t - h_t - I_t - (W_t - W_{t-1}). \tag{13}$$

Therefore, the return on consumption claims, $R_{C,t+1}$, is

$$R_{C,t+1} = \frac{P_{C,t+1} + C_{t+1}}{P_{C,t}}, \tag{14}$$

where $P_{C,\,t}$ is the price of consumption claims at time t.

The representative agent chooses the optimal inventories and consumption profiles, while the firm operates under its optimal investment policy. The agent's problem can be summarized as follows:

$$\begin{aligned} U_t &= U(K_{t-d}, \ldots, K_t, S_{t-h}, \ldots, S_{t-1}, W_{t-1}, Z_t) \\ &= \max_{\{C_t, S_t, W_t\}} \left\{ (1-\beta) C_t^{\frac{1-\gamma}{\theta}} + \beta [\mathbb{E}_t U_{t+1}^{1-\gamma}]^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \end{aligned} \tag{15}$$

$$s.t. \ K_{t+1} = K_t + g_t - \delta \sum_{i=0}^{d} u_i K_{t-i}, \tag{16}$$

$$C_t = Z_t^{1-\alpha} \left[K_{t-d}^{1-\omega} W_{t-1}^{\omega} \right]^{\alpha} - \sum_{i=0}^{h} w_i S_{t-i} - h_t - (W_t - W_{t-1}). \tag{17}$$

To distinguish from the standard production model, the TTB-only model, and a model with TTB and TTP, it is instructive to look at the state variables in these models. The standard production model uses current capital stock, the productivity shock, and inventories at the end of the previous period, $\{K_t, Z_t, W_{t-1}\}$, as state variables. The TTB constraint expands the state space by introducing historically initiated projects, i.e., $\{S_{t-h}, \ldots, S_{t-1}, K_t, Z_t, W_{t-1}\}$. Given the capital stock dynamics, the state space is equivalent to $\{K_t, K_{t+1}, \ldots, K_{t+h}, Z_t, W_{t-1}\}$. Incorporating the TTP constraint in addition to the TTB constraint adds historical capital stocks to the state space, i.e., $\{K_{t-d}, \ldots, K_{t-1}, K_t, K_{t+1}, \ldots, K_{t+h}, Z_t, W_{t-1}\}$. All of these state variables contribute to the firm value.

2.3. The equilibrium conditions

The Lagrangian function of the maximization problem is

$$\begin{aligned} L_t = & \left\{ (1-\beta) C_t^{\frac{1-\gamma}{\theta}} + \beta [\mathbb{E}_t U_{t+1}^{1-\gamma}]^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \\ & + \xi_t \left[K_t + g_t - \delta \sum_{i=0}^{d} u_i K_{t-i} - K_{t+1} \right] \\ & + \mu_t \Bigg[Z_t^{1-\alpha} [K_{t-d}^{1-\omega} W_{t-1}^{\omega}]^{\alpha} \\ & - \sum_{i=0}^{h} w_i S_{t-i} - h_t - (W_t - W_{t-1}) - C_t \Bigg], \end{aligned} \tag{18}$$

where $\{\xi_t, \mu_t\}$ are the current value Lagrangian multipliers associated with constraints (16)–(17), respectively.

The first-order condition with respect to C_t gives

$$\mu_t = (1-\beta) U_t^{\frac{1}{\psi}} C_t^{-\frac{1}{\psi}}, \tag{19}$$

which defines the marginal utility of consuming one additional unit of good at time t.

Similarly, the optimal inventory policy satisfies the following first-order condition:

$$\frac{\partial L_t}{\partial W_t} = \mathbb{E}_t \left[\frac{\partial U_t}{\partial U_{t+1}} \frac{\partial U_{t+1}}{\partial W_t} \right] - \mu_t \left[\frac{\partial h_t}{\partial W_t} + 1 \right] = 0. \tag{20}$$

Applying the envelope condition to (20), we obtain

$$\begin{aligned} & \mathbb{E}_t[M_{t,t+1}] \\ & = -\mathbb{E}_t \left[M_{t,t+1} \alpha \omega Z_{t+1}^{1-\alpha} (K_{t-d+1}^{1-\omega} W_t^{\omega})^{\alpha-1} K_{t-d+1}^{1-\omega} W_t^{\omega-1} \right] \\ & \quad + \frac{\partial h_t}{\partial W_t} + 1. \end{aligned} \tag{21}$$

This equation captures the mean of the stochastic discount factor, which is tied to inventories. Thus, inventories are linked to the risk-free rate.

The optimal capital stock at time $t+1$ satisfies

$$\frac{\partial L_t}{\partial K_{t+1}} = \mathbb{E}_t \left[\frac{\partial U_t}{\partial U_{t+1}} \frac{\partial U_{t+1}}{\partial K_{t+1}} \right] - \xi_t = 0, \tag{22}$$

where ξ_t is the marginal utility at time t of increasing one additional unit of capital stock at time $t+1$. For simplicity, define

$$q_t \equiv \frac{\xi_t}{\mu_t}, \tag{23}$$

which is the marginal q usually defined in the production models.

Applying the envelope conditions recursively, we obtain the evolution of marginal q as follows:

$$\begin{aligned} q_t = & \ \mathbb{E}_t[M_{t,t+1} q_{t+1}] - \delta \mathbb{E}_t \left[\sum_{i=1}^{d+1} M_{t,t+i} u_{i-1} q_{t+i} \right] \\ & + \mathbb{E}_t \Bigg\{ M_{t,t+d+1} \Bigg[\alpha (1-\omega) K_{t+1}^{-\omega} W_{t+d}^{\omega} Z_{t+d+1}^{1-\alpha} \left[K_{t+1}^{1-\omega} W_{t+d}^{\omega} \right]^{\alpha-1} \\ & + q_{t+d+1} \frac{\partial g_{t+d+1}}{\partial K_{t+1}} - \frac{\partial h_{t+d+1}}{\partial K_{t+1}} \Bigg] \Bigg\}. \end{aligned} \tag{24}$$

So, current q is related to future q, $\{q_{t+1}, \ldots, q_{t+d+1}\}$, through the depreciation channel caused by the TTP feature.

The optimal capital investment policy satisfies

$$\mathbb{E}_t \left[M_{t,t+h} q_{t+h} \frac{\partial g_{t+h}}{\partial S_t} \right] = \mathbb{E}_t \left[\sum_{i=0}^{h} M_{t,t+i} w_i \right]. \tag{25}$$

The above condition presents the marginal benefits and costs at time t of adding one additional unit of capital stock at time $t+h+1$.

2.4. The numerical solution

Three sources contribute to the computational complexity of this model: (a) the number of state variables (e.g.,

Table 1
Parameters.
This table summarizes the parameters used in the calibrations. The time unit is a quarter.

Parameters	Description	Value
Fixed parameters		
α	Elasticity of capital	0.358
δ	Depreciation rate of capital	0.027
ω	Elasticity of inventory	0.16
ρ	Persistence of the technology shock	0.95
η	Inventory cost coefficient	0.08
τ	Curvature of the inventory holding costs	0.5
$\{w_0, w_1, w_2\}$	Proportions of a project invested ($h = 2$)	{0.1, 0.1, 0.8}
$\{u_0, u_1, u_2\}$	Proportions of capital stock depreciation ($d = 2$)	{1/3, 1/3, 1/3}
Calibrated parameters		
σ	Volatility of the technology shock	0.018
χ	Curvature of the capital adjustment costs	0.73
β	Time discount	0.987
γ	Relative risk aversion	7.5
ψ	Elasticity of intertemporal substitution	0.03

seven state variables in the main model); (b) the delays in production, which make the equilibrium conditions more deeply recursive (see Eq. (24));[3] and (c) the recursive preferences, which complicate computing the pricing kernel. I solve this model by a projection method with Galerkin weighted residuals. Projection methods are highly accurate and widely used in the literature (see, e.g., Aruoba, Fernández-Villaverde, and Rubio-Ramírez, 2006). I approximate the variables of interest with a complete set of Chebyshev polynomials. Given the number of state variables in the model, it is numerically challenging to compute the multidimensional integration for the conditional expectations and Galerkin weighted residuals. To avoid the curse of dimensionality, I apply non-product monomial rules instead of a full tensor grid, where the number of grids only grows quadratically with the number of state variables. These rules are derived by Hammer and Stroud (1958) and Stroud and Secrest (1963) (see also Judd, 1998). Pichler (2011) demonstrates the accuracy and effectiveness of this method in a multi-country RBC model. The polynomial coefficients are solved from a system of nonlinear equations by a trust region approach, provided in Intel Math Kernel Library. Internet Appendix A summarizes the technical details of numerical computing and addresses some specific computational issues in this model.

3. Calibration approach

3.1. Data

The model is calibrated to US data over 1964–2012. I obtain annual market return and the risk-free rate data from the annual Fama-French factor data file from Kenneth French's website. The real returns are adjusted by the consumer price index (CPI) from the National Income and Product Accounts (NIPA) Table 2.3.4. As there is no leverage in this model, for comparison, I compute the unlevered market returns, assuming a debt-to-equity ratio of 0.5 (see, e.g., Barro, 2006). The real output is measured as the real gross domestic product from the NIPA Table 1.1.6. The real consumption is defined as real nondurable goods and services, computed from the NIPA Tables 2.3.4 and 2.3.5. The investment is computed as the sum of the real gross private domestic investment (excluding the subcategory of change in private inventories), the government gross investment adjusted by the government gross investment price index, and the personal consumption expenditures on durable goods adjusted by the durable goods price index, computed from the NIPA Tables 1.1.6, 3.9.4, 3.9.5, 2.3.4, 2.3.5, 5.7.6A, and 5.7.6B. All these quantities are quarterly and normalized by the civilian noninstitutional population with age over 16, which is from the Current Population Survey (Serial ID LNU00000000Q). The nominal capital is measured as the fixed assets from the NIPA Table 5.10. The nominal inventory refers to the private inventories (from the NIPA Tables 5.8.5A and 5.8.5B). The nominal consumption is the personal consumption expenditures from the NIPA Table 1.1.5. The nominal output is GDP from the NIPA Table 1.1.5. Subject to data availability, these nominal data are annual only. These nominal data are used to compute the ratios of inventory/capital, output/capital, and consumption/capital. The key moments of quarterly macroeconomic quantities and annual asset prices are reported in Table 2. The volatilities of output, consumption, and investment are computed from the Hodrick-Prescott filter.

3.2. Parameters

The parameters chosen are close to those in the literature and are summarized in Table 1. The main model assumes a 3-quarter TTB ($h = 2$) and a 3-quarter TTP ($d = 2$). The choice of h and d are based on empirical estimation in Internet Appendix B and can be understood from the following empirical facts. First, asset prices lead macroeconomic quantities by about two quarters (Backus, Routledge, and Zin, 2007, 2010). As demonstrated in Section 4.6.1, this suggests a 3-quarter TTB ($h = 2$). Second, without distinguishing TTB and TTP, Christiano and Vigfusson (2003) estimate a model with a production delay of four quarters. Hence, I set $d = 2$ in the main model. The elasticity of

[3] Effectively, this recursive nature makes the model similar to a system of 21 state variables in terms of numerical computing complexity.

capital (α) is 0.358, which is similar to that in Boldrin, Christiano, and Fisher (2001) and Kaltenbrunner and Lochstoer (2010). The quarterly depreciation rate (δ) is 0.027, which is the average investment/capital ratio over 1964–2012 from the NIPA tables. The persistence of the technology shock (ρ) is 0.95. The volatility of the technology shock (σ) is 0.018, which is set to fit the output volatility and similar to that in Boldrin, Christiano, and Fisher (2001).[4] Christiano and Vigfusson (2003) suggest that most investment expenditures occur in the later periods, so I set the proportions of a project invested as $\{w_0, w_1, w_2\} = \{0.1, 0.1, 0.8\}$ in the main model. The proportions of capital stock depreciation are set as $\{u_0, u_1, u_2\} = \{1/3, 1/3, 1/3\}$.

The time discount (β), the curvature of capital adjustment costs (χ), the EIS (ψ), and the relative risk aversion (γ) are calibrated to match the asset prices. I set $\beta = 0.987$. The curvature of capital adjustment costs (χ) is set to 0.73, which is in the range examined in the literature.[5] Since the technology frictions are temporary and shocks are transitory in this model, a small EIS is necessary to generate a sizable risk premium. The chosen EIS (ψ) is 0.03, which is similar to that in Gomes, Kogan, and Yogo (2009). Relative risk aversion (γ) is set to 7.5.

Three parameters remain, and all of these are related to inventories: the elasticity of inventory (ω), the inventory cost coefficient (η), and the curvature of inventory holding costs (τ). Since these parameters are new and there is no existing literature to provide guidance, I use the steady state equations to set these three parameters to match the means of the inventory/capital ratio, the output/capital ratio, and the consumption/capital ratio computed from the NIPA tables over 1964–2012, which are 7.03%, 37.06%, and 23.56%, respectively. This gives $\tau = 0.5,\ \eta = 0.08,$ and $\omega = 0.16$.

3.3. Calibration

To investigate the roles of inventories, TTB, and TTP, I first consider six benchmark models in which these features are added gradually. These are the standard RBC model (Benchmark 1), the standard RBC model with inventory (Benchmark 2), the standard RBC model with a 3-quarter ($h = 2$) time-to-build constraint (Benchmark 3), the standard RBC model with inventory and a 3-quarter ($h = 2$) time-to-build constraint (Benchmark 4), the standard RBC model with a 3-quarter ($d = 2$) time-to-produce constraint (Benchmark 5), and the standard RBC model with inventory and a 3-quarter ($d = 2$) time-to-produce constraint (Benchmark 6). The main model includes all features; it is the standard RBC model with inventory, a 3-quarter ($h = 2$) TTB, and a 3-quarter ($d = 2$) TTP. All models include capital adjustment costs.

Similar to Aruoba, Fernández-Villaverde, and Rubio-Ramírez (2006), I simulate each model for 1,000 paths. Starting from the non-stochastic steady state, each path has 300 periods, and the first 100 periods are a burn-in to eliminate the transition from the deterministic steady state to the ergodic distribution. One unit of time represents a quarter, so each path is 50 years long, which is roughly identical to the length of the empirical sample.

3.4. Numerical accuracy

Before discussing the calibration results, I first evaluate the numerical accuracy of the projection method employed. For comparison, I also compute the model by another popular algorithm, the perturbation method (e.g., first-order, second-order, and third-order perturbations). Following (Petrosky-Nadeau and Zhang, 2014), I compute the interpretable Euler equation errors, both static and dynamic. The static consumption error, e_C^S, is defined as follows:

$$e_C^S = \frac{C_t - C_t^{implied}}{C_{ss}}, \tag{26}$$

where C_{ss} is the steady state value of consumption, C_t is computed from a given numerical method (e.g., a projection or perturbation method), and $C_t^{implied}$ is the implied consumption value. The implied consumption value is computed from the Euler equation as

$$C_t^{implied} = \left\{ \mathbb{E}_t \left[\beta C_{t+1}^{-\frac{1}{\psi}} \left[\frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_t U_{t+1}^{1-\gamma}} \right]^{1-\frac{1}{\theta}} (P_{t+1} + C_{t+1}) \right] \Big/ P_t \right\}^{-\psi}, \tag{27}$$

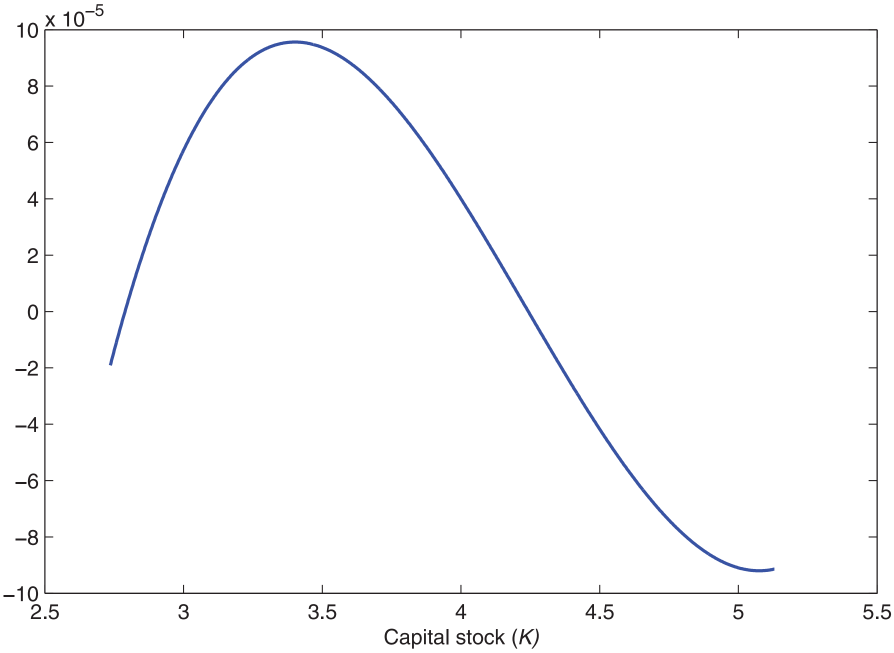
where the conditional expectation is computed accurately from the Gauss-Hermite quadrature with 20 grid points for the shocks. If the numerical method is accurate, then e_C^S should be zero.

Fig. 1 reports the static consumption errors of the main model, computed from the projection method and the perturbation methods for capital over $[80\%K_{ss}, 150\%K_{ss}]$, where K_{ss} is the steady state value of the capital stock. Panel (a) shows that the projection method is highly accurate. The static consumption errors are in the magnitude of 10^{-5}; i.e., the errors are less than 0.001% of the steady state value. In contrast, Panel (b) shows that the static consumption errors from the perturbation methods have a magnitude of 10^{-3}. The first-order and third-order perturbations are very close and are slightly better than the second-order perturbation.

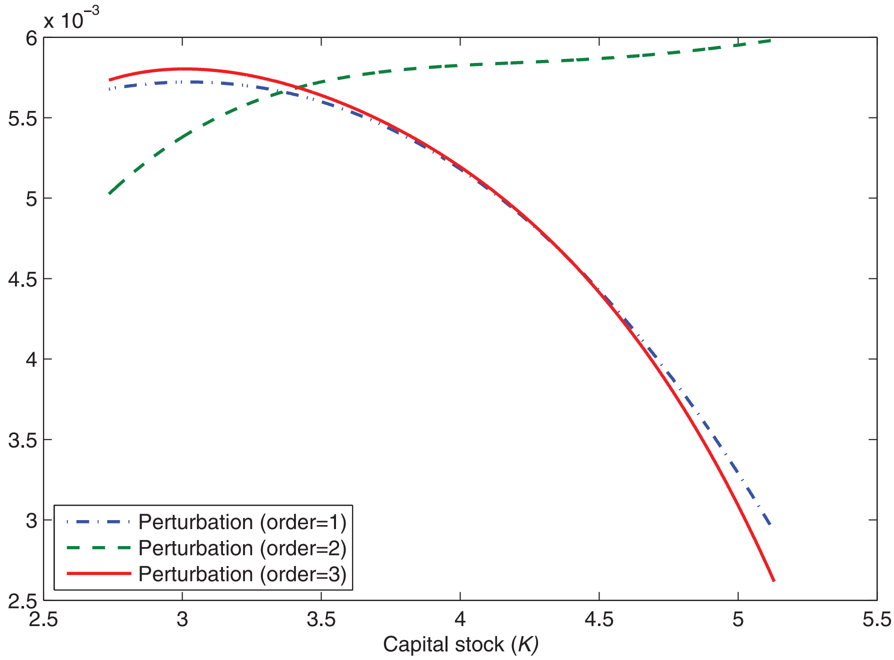
The static errors measure only one-period-ahead approximation errors. To investigate the accumulation of approximation errors over time in a dynamic setting, I follow Petrosky-Nadeau and Zhang (2014) to compute the dynamic errors. Specifically, I first generate time series for C_t, using a given numerical method (e.g., a projection or perturbation method). Next, I generate an alternative series using the following steps: (1) In each period, compute the implied consumption (C_t^D), where the conditional expectation is computed accurately from the Gauss-Hermite

[4] This shock volatility is in the high end of values typically used in the literature, which range from 0.7% to 4.1%, as the model lacks a propagation mechanism.

[5] For example, Jermann (1998) and Boldrin, Christiano, and Fisher (2001) choose 0.23, and Kaltenbrunner and Lochstoer (2010) set it from 0.7 to 18.



(a) Consumption errors, projection



(b) Consumption errors, perturbation

Fig. 1. The static Euler equation error. Panels (a) and (b) plot the static consumption errors from the Euler equation of the main model, for capital stock K_t over [80%K_{ss}, 150%K_{ss}], where K_{ss} is the steady state value of the capital stock. The consumption error is defined as a fraction of its steady state value. Panel (a) displays the static consumption errors of the projection method. Panel (b) compares the static consumption errors of the perturbation method with different orders.

Table 2
Calibrations: Different models.
This table summarizes key moments of macroeconomic quantities and asset prices from calibrations of six benchmark models and the main model. Benchmark models are the standard RBC model (Benchmark 1), the standard RBC model with inventory (Benchmark 2), the standard RBC model with a 3-quarter ($h = 2$) time-to-build constraint (Benchmark 3), the standard RBC model with inventory and a 3-quarter ($h = 2$) time-to-build constraint (Benchmark 4), the standard RBC model with a 3-quarter ($d = 2$) time-to-produce constraint (Benchmark 5), and the standard RBC model with inventory and a 3-quarter ($d = 2$) time-to-produce constraint (Benchmark 6). The main model is the standard RBC model with inventory, a 3-quarter ($h = 2$) time-to-build constraint, and a 3-quarter ($d = 2$) time-to-produce constraint. All models include capital adjustment costs. The empirical data are from the NIPA tables and the annual Fama-French factors over 1964–2012. The numbers in parentheses are unlevered market returns, assuming a debt-to-equity ratio of 0.5 (see, e.g., Barro, 2006). The macroeconomic quantities are reported as quarterly, while the asset prices are annualized. The volatilities of output, consumption, and investment are computed from the Hodrick-Prescott filter. All moments are reported in percentages, except the Sharpe ratio.

	U.S. data (1964–2012)	Benchmark (1) No inventory ($h = 0, d = 0$)	Benchmark (2) Inventory ($h = 0, d = 0$)	Benchmark (3) No inventory ($h = 2, d = 0$)	Benchmark (4) Inventory ($h = 2, d = 0$)	Benchmark (5) No inventory ($h = 0, d = 2$)	Benchmark (6) Inventory ($h = 0, d = 2$)	Main model Inventory ($h = 2, d = 2$)
Panel A: Macroeconomic quantities								
Volatility of output								
$\sigma(Y)$	1.55	1.46	1.60	1.46	1.81	1.39	1.47	1.56
Volatility of consumption								
$\sigma(C)$	0.84	0.58	0.55	1.76	0.88	0.79	0.66	0.80
Volatility of investment								
$\sigma(I)$	5.11	4.07	5.13	0.99	5.48	2.57	3.10	3.46
Mean and volatility of the inventory/consumption ratio								
W/C	30.03		10.62		10.31		27.68	27.83
$\sigma(W/C)$	7.92		1.04		1.78		2.95	3.62
Mean and volatility of the inventory/capital ratio								
W/K	7.03		1.14		1.19		6.28	7.26
$\sigma(W/K)$	1.65		0.17		0.25		0.90	1.41
Panel B: Asset prices								
Mean and volatility of the risk-free rate								
$E[R_f]$	1.57	3.63	3.80	1.52	1.60	0.98	0.91	1.43
$\sigma(R_f)$	2.18	1.42	1.40	17.81	5.92	1.36	1.93	2.42
Mean and volatility of the return on dividend claims								
$E[R_D]$	7.51 (5.53)	5.09	4.35	6.47	2.66	2.89	1.70	5.28
$\sigma(R_D)$	18.04 (12.03)	9.92	11.69	19.12	18.37	7.47	7.20	10.11
Risk premium of the return on dividend claims								
$E[R_D - R_f]$	5.94 (3.96)	1.46	0.55	4.95	1.06	1.91	0.79	3.85
Sharpe ratio of dividend claims								
$E[R_D - R_f]/\sigma(R_D)$	0.33	0.15	0.05	0.26	0.06	0.26	0.11	0.38
Mean and volatility of the return on consumption claims								
$E[R_C]$		5.69	5.82	11.64	8.93	7.67	6.89	7.47
$\sigma(R_C)$		13.16	11.50	26.64	24.62	23.99	23.52	22.91
Risk premium of the return on consumption claims								
$E[R_C - R_f]$		2.06	2.02	10.12	7.33	6.69	5.98	6.04
Sharpe ratio of consumption claims								
$E[R_C - R_f]/\sigma(R_C)$		0.16	0.18	0.38	0.30	0.28	0.25	0.26

quadrature with 20 grid points for the shocks. (2) Compute the new project size from this implied consumption and the resource constraint. (3) Use this new project size as the input for approximation in the next period. The dynamic consumption error, e_C^D, is defined as:

$$e_C^D = \frac{C_t - C_t^D}{C_{ss}}. \tag{28}$$

As in the calibration exercises described earlier, I simulate the model for 1,000 paths. Starting from the non-stochastic steady state, each path has 300 periods, and the first 100 periods are a burn-in. Fig. 2 shows the empirical cumulative distribution of the dynamic consumption errors. Panel (a) shows that the dynamic consumption errors from the projection method are very small (the largest errors are in the magnitude of 10^{-3}). In contrast, the perturbation methods exhibit very large dynamic consumption errors in Panel (b). For example, most dynamic consumption errors fall between 6% and 10%.

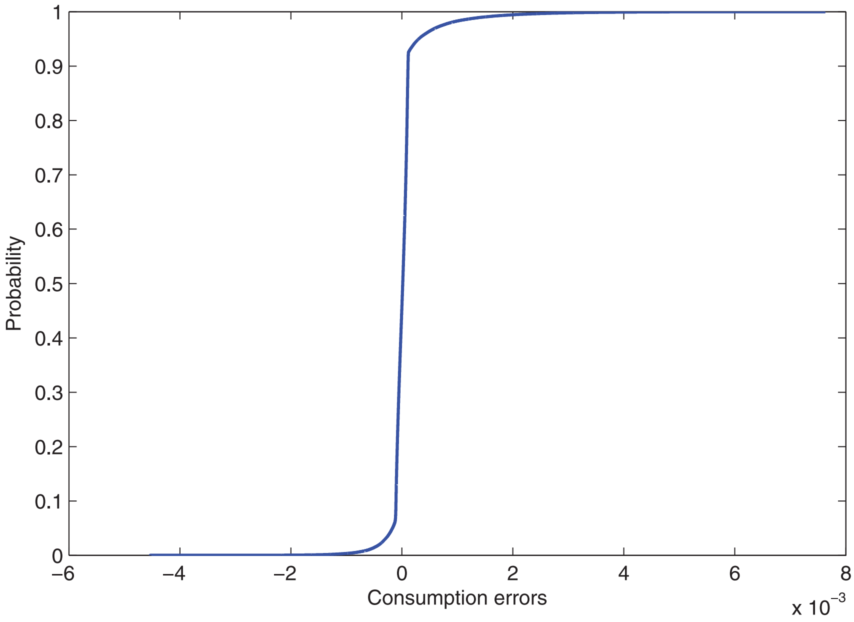
In short, these accuracy tests confirm that the projection method is highly accurate. Both the static and dynamic consumption errors are at least an order of 2 smaller than those from the perturbation methods. Perturbation methods with different orders are quite close, but the second-order method is slightly worse than the first-order and third-order perturbations.[6]

4. Results

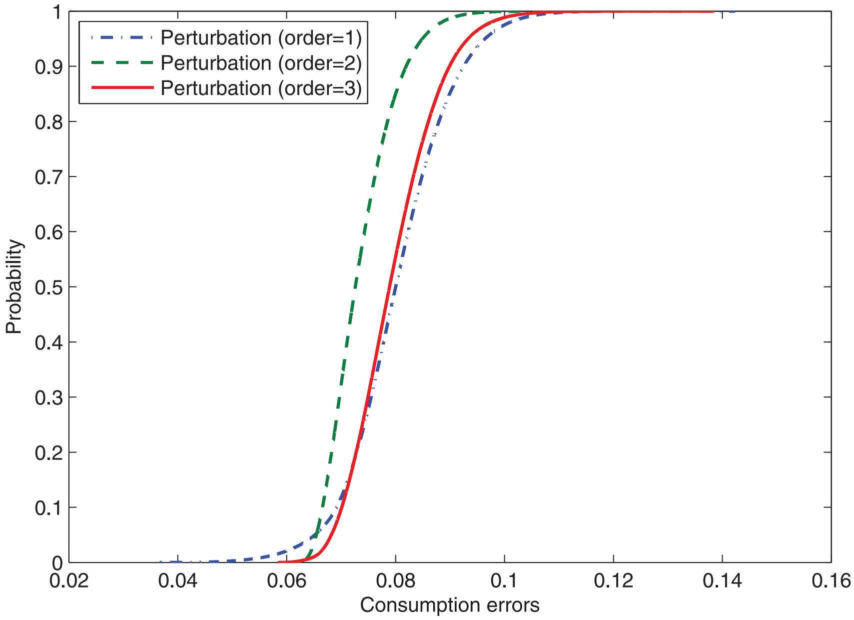
4.1. Main results

Table 2 reports simulation results of the main model and six other benchmark models. As in the standard RBC model, it is not surprising to see that the model can reasonably match the volatilities of quantities like output and consumption. Only the volatility of investment is a little

[6] Internet Appendix C further compares the policy and value functions for the first-order, second-order, and third-order perturbation methods. It shows that the second-order perturbation generates a counterfactual approximation of the utility function in this model. Overall, the third-order perturbation seems to be the best among the perturbation methods.



(a) Cumulative distribution of consumption errors, projection



(b) Cumulative distribution of consumption errors, perturbation

Fig. 2. The dynamic Euler equation error. Panels (a) and (b) plot the dynamic consumption errors from the Euler equation of the main model. The consumption error is defined as a fraction of its steady state value. Each model is simulated for 1,000 paths. Starting from the non-stochastic steady state, each path has 300 periods, and the first 100 periods are a burn-in. The empirical cumulative distribution functions for the dynamic errors are plotted. Panel (a) displays the dynamic consumption errors of the projection method. Panel (b) compares the dynamic consumption errors of the perturbation method with different orders.

lower than that in the data. This is due to the capital adjustment costs introduced in the model, as observed in the literature (e.g., Boldrin, Christiano, and Fisher, 2001). Also, under TTB and TTP constraints, firms adjust investment more often via inventories than via capital, as the former is less costly and more responsive. Inventory ratios are also in line with the data, except that the volatility of inventory/consumption ratio is lower than that in the data. The moments of asset prices are close to those in the data as well. For example, the mean and volatility of the return on dividend claims are 5.28% and 10.11%, while they are 5.53% and 12.03% in the data for the unlevered returns, respectively. As the model aims to fit the consumption volatility, for the sake of completeness, Table 2 also reports the return on consumption claims. The main model generates a return on consumption claims of 7.47% with a volatility of 22.91%, both of which are close to the levered stock returns observed in the data.[7]

More importantly, the main model can generate a risk-free rate with a low volatility of 2.42%. Excessively volatile risk-free rates have been a challenge for production-based models when EIS is small (Boldrin, Christiano, and Fisher, 2001; Jermann, 1998; Kaltenbrunner and Lochstoer, 2010). In the main model, inventory investment is less risky than capital investment, and this helps to generate a less volatile risk-free rate process.

Six benchmark models in Table 2 differentiate the contributions of various features in the model, namely, inventories, TTB, and TTP. First, we see that simply adding inventories to a standard RBC model in Benchmark 2 does not help the fit of macroeconomic quantities and asset prices. In fact, the risk premium of the dividend claims decreases from 1.46% in Benchmark 1 to 0.55% in Benchmark 2, as inventories smooth consumption. Comparing Benchmarks 1 and 3, we see that introducing TTB makes the consumption process extremely volatile while investment is too stable when most investment expenditures happen in the later periods, because investment is very risky. For example, the consumption volatility is as high as 1.76% in Benchmark 3, compared with 0.58% in Benchmark 1. As a result, asset prices are very volatile, e.g., the risk-free rate has a volatility of 17.81%. The good news is that TTB in Benchmark 3 generates a risk premium of 4.95% to the dividend claims together with a Sharpe ratio of 0.26. As shown later, such a sizable risk premium comes from the procyclical payouts generated by the TTB feature when capital expenditure occurs mostly in the later periods. Benchmark 4 shows that inventory smooths consumption significantly (with a consumption volatility of 0.88%). As a result, inventory also decreases the risk premium sharply to 1.06% and the Sharpe ratio to 0.06. Also, as the inventory holdings under TTB in Benchmark 4 are too small relative to the data, consumption remains more volatile under TTB in Benchmark 4, compared with a standard RBC model in Benchmark 2. As a result, asset prices are still very volatile (e.g., the risk-free rate has a volatility of 5.92%).

Regarding the TTP constraint, Benchmark 5 shows that TTP increases the consumption volatility to 0.79%, compared with 0.58% in Benchmark 1. The volatile consumption translates into a higher risk premium of 1.91% and a higher Sharpe ratio of 0.26, together with a low volatility risk-free rate. Another impact of TTP can be seen from investment volatility in Benchmark 5. As capital investment is much riskier under the TTP constraint, it becomes less volatile, compared with Benchmark 1. One might wonder if we can interchange TTB and TTP. Comparing Benchmarks 3 and 5, we see that TTP generates a smaller risk premium and a much less volatile risk-free rate, together with moderate consumption and investment volatilities. Adding inventory to the TTP constraint, Benchmark 6 shows that TTP is necessary to match the moments of inventory observed in the data. The reason is that when facing a productivity shock, firms can adjust output via productive capital quickly if there is no TTP constraint. Hence, inventories are less important in this case. However, with the TTP constraint, since the productivity of current capital stock is unobservable, capital investments are unable to effectively smooth out the consumption volatility introduced by TTP. Thus, agents have to rely heavily on inventories to smooth consumption in the presence of TTP. Again, the negative side of inventory is that the risk premium shrinks to 0.79% in Benchmark 6. Combining TTB with TTP, the main model produces a larger risk premium of 3.85% (compared with 1.06% in the TTB-only model of Benchmark 4), because the TTP feature amplifies the procyclical payouts generated by the TTB feature (see further discussion in Section 4.2).

None of the Benchmarks can fit both macroeconomic quantities and asset prices. Only putting TTB, TTP, and inventory together in the main model can reasonably match both macroeconomic quantities and asset prices. Table 2 illustrates that although both TTB and TTP are helpful in generating a sizable equity premium from procyclical payouts, TTB also produces an excessively volatile consumption process and asset prices. Inventory is necessary to smooth consumption under TTB. Only the TTP feature ensures substantial inventory holdings. Inventory helps smooth consumption and reduce the excessive asset price volatilities. TTP also amplifies the procyclical payouts generated by the TTB constraint.

4.2. Sources of equity risks

Given the sizable equity premium generated in the main model, it is still necessary to verify that this is indeed a risk premium for volatile equity payouts. In fact, as demonstrated in Kaltenbrunner and Lochstoer (2010), a common problem with RBC models is that payouts are countercyclical. The reason is that investments respond to technology shocks too strongly in these models even in the presence of high capital adjustment costs; i.e., investments are highly procyclical and more volatile than output, which implies countercyclical payouts. This counterfactual feature leads to the failure to generate a sizable equity premium from equity payouts, since claims to the payouts provide hedging (e.g., Kaltenbrunner and Lochstoer, 2010). However, this is not the case in the main model, as the TTB feature, along with larger investment expenditures in the later periods, slows the response of investments to technology shocks. Fig. 3 compares the impulse responses of payouts in several models.[8] As already shown in Kaltenbrunner and Lochstoer (2010),

[7] One should interpret these results with caution. In the model, the consumption claims apply to the wealth portfolio which includes wages, while human capital is missing in the stock market data. I am grateful to the referee for pointing this out.

[8] Internet Appendix D provides impulse responses of other macroeconomic quantities.

Table 3
Return predictability.
This table presents the predictability regressions of future stock returns or dividend growth rates on the dividend-price ratio. The annual U.S. market returns (including NYSE/Amex/Nasdaq) from CRSP over 1925–2012, deflated by the CPI, are used. The simulated data from six benchmark models and the main model are firstly aggregated annually, and the median values of regressions over the 1,000 sample paths are reported. See Table 2 for a detailed description of these models. The *t*-statistics are corrected for heteroskedasticity and autocorrelation, using Newey-West standard errors.

	Panel A: Regression: $R_{t,t+k} = a + b\frac{D_t}{P_t} + \varepsilon_{t+k}$						Panel B: Regression: $\frac{D_{t+k}}{D_t} = a + b\frac{D_t}{P_t} + \varepsilon_{t+k}$					
	k=1			k=2			k=1			k=2		
	b	$t(b)$	R^2	b	$t(b)$	R^2	b	$t(b)$	R^2	b	$t(b)$	R^2
U.S. data	3.38	2.41	0.06	6.47	2.86	0.10	−0.13	−0.12	0.00	−0.65	−0.37	0.00
Benchmark (1)	4.12	2.53	0.11	8.14	3.25	0.21	−0.93	−1.15	0.03	−2.68	−2.02	0.11
Benchmark (2)	4.20	2.74	0.13	8.54	3.52	0.24	−1.58	−1.08	0.04	−4.33	−1.64	0.10
Benchmark (3)	9.89	2.55	0.12	14.55	2.79	0.17	0.89	0.96	0.02	−0.96	−0.70	0.02
Benchmark (4)	18.64	1.59	0.05	32.31	1.86	0.09	−3.12	−0.78	0.01	−14.63	−2.34	0.11
Benchmark (5)	1.74	1.90	0.08	3.54	2.54	0.16	−1.98	−0.70	0.03	−4.45	−0.74	0.04
Benchmark (6)	1.93	4.71	0.30	3.87	5.97	0.49	−0.09	−0.05	0.01	−0.19	−0.06	0.01
Main model	2.00	3.71	0.22	4.25	4.85	0.39	−0.14	−0.24	0.01	−0.36	−0.22	0.01

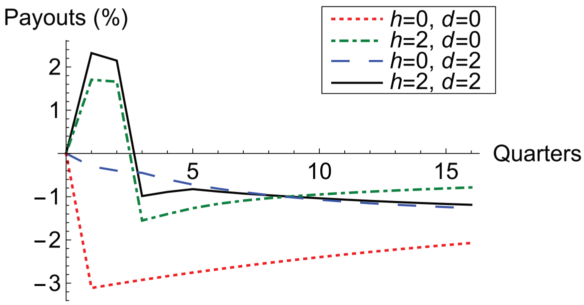


Fig. 3. Impulse response function of payouts. This figure depicts the impulse response of payouts after a positive, one-standard-deviation technology shock at time 1, plotted as a percentage deviation from its steady state value, from four benchmark models and the main model ($h = 2, d = 2$). Benchmark models include Benchmark 1 ($h = 0, d = 0$), Benchmark 4 ($h = 2, d = 0$), and Benchmark 6 ($h = 0, d = 2$). See Table 2 for a detailed description of these models.

payouts respond negatively after a positive technology shock in the standard RBC model (Benchmark 1) and in Benchmark 6 ($h = 0, d = 2$). However, payouts respond positively in the first two periods in the presence of a TTB feature, as shown in the main model and in Benchmark 4 ($h = 2, d = 0$). This can be induced from Panel (e) of Fig. E.1 in Internet Appendix D, in which total investments respond very moderately in the first two periods when TTB is present. Thus, TTB helps generate procyclical payouts, which contribute to the sizable equity premium (e.g., 1.06% in Benchmark 4). This mechanism is different from that in Petrosky-Nadeau, Zhang, and Kuehn (2013), who use labor market frictions to generate procyclical payouts.

In addition, although TTP alone cannot generate procyclical payouts, it amplifies the procyclical payouts generated by TTB in Fig. 3. For example, after a positive, one-standard-deviation technology shock at time 1, the payouts increase by 2.34% in the main model, as compared with a 1.70% increase in the TTB-only model (Benchmark 4). The reason is that investment becomes much riskier and hence less procyclical under the TTP constraint (see Table 2). More procyclical payouts contribute to the larger equity premium found in the main model. Relative to Benchmark 4, TTP adds 2.79% to the risk premium of dividend claims in the main model.

Another concern over the recursive preferences with a small EIS is that the equity premium might be driven mainly by the term premium. When EIS is small, agents are extremely averse to the intertemporal consumption variations, which produces a large term premium embedded into the equity premium. For example, in Kaltenbrunner and Lochstoer (2010), term premium is close to the return on consumption claims. Numerically, the term premium of a 10-year zero-coupon bond over a 1-quarter zero-coupon bond is only 1.95% in the main model, which is much smaller than the risk premium of dividend claims (3.85%) or the risk premium of consumption claims (6.04%). Thus, the equity premium implied by the main model is largely contributed by the procyclical payouts, rather than the term premium. The main model avoids an abnormally large term premium even when EIS is small because the risk-free rate is not excessively volatile.

4.3. Return predictability

Although suffering from measurement and econometric methodology problems, previous studies often find that the dividend-price ratio can predict future returns (see Cochrane, 2008, for a summary). Here, I perform similar predictability regressions to investigate the conditional asset pricing implications of the model, in addition to the unconditional moments reported before. Table 3 reports predictability regressions of the dividend-price ratio. The data are the NYSE/Amex/Nasdaq annual market returns over 1925–2012 obtained from the Center for Research in Security Prices (CRSP), deflated by the CPI. The regression results are similar to those in Cochrane (2008). That is, returns can be predicted by the dividend-price ratio, while dividend growth is unpredictable. The return predictability increases with the horizon as these variables are persistent.

I run similar regressions over each sample path of the simulated data generated from the six benchmark models and the main model. The median values of the regressions are reported. All models show the return predictability in

Table 4
Stock returns and investment returns.
This table compares stock returns and investment returns implied by six benchmark models and the main model. See Table 2 for a detailed description of these models. Investment returns are defined as the returns on the productive capital, or the value-weighted average returns on the productive capital and inventory investment if inventory is present. All moments are annualized and reported in percentages.

	Benchmark (1) No inventory ($h=0$, $d=0$)	Benchmark (2) Inventory ($h=0$, $d=0$)	Benchmark (3) No inventory ($h=2$, $d=0$)	Benchmark (4) Inventory ($h=2$, $d=0$)	Benchmark (5) No inventory ($h=0$, $d=2$)	Benchmark (6) Inventory ($h=0$, $d=2$)	Main model Inventory ($h=2$, $d=2$)
Panel A: Asset prices							
Mean and volatility of the return on dividend claims							
$E[R_D]$	5.09	4.35	6.47	2.66	2.89	1.70	5.28
$\sigma(R_D)$	9.92	11.69	19.12	18.37	7.47	7.20	10.11
Risk premium of the return on dividend claims							
$E[R_D - R_f]$	1.46	0.55	4.95	1.06	1.91	0.79	3.85
Panel B: Asset prices implied by investment							
Mean and volatility of the investment returns							
$E[R_I]$	5.12	4.93	7.25	5.85	2.92	2.45	4.19
$\sigma(R_I)$	9.18	10.27	19.4	16.04	6.6	6.86	9.45
Risk premium implied by the investment returns							
$E[R_I - R_f]$	1.49	1.13	5.73	4.25	1.94	1.54	2.76
Contribution to the return on dividend claims							
$E[R_I]/E[R_D]$	101%	113%	112%	220%	101%	144%	79%
$\sigma(R_I)/\sigma(R_D)$	93%	88%	101%	87%	88%	95%	93%

Panel A. However, Benchmarks 1 and 2 show marginal significance of predicting two-year-ahead dividend growth in Panel B. With the TTB constraint, Benchmarks 3 and 4 produce much larger regression coefficients in Panel A, compared with those in Benchmarks 1 and 2. In addition, Benchmark 4 shows that two-year-ahead dividend growth is predictable. Adding the TTP constraint in Benchmarks 5 and 6 generates regression coefficients that are too small in Panel A, compared with those in the data. Adding inventory increases the regression coefficients in Panel A. Putting all features together, the main model reasonably matches the predictability of the dividend-price ratio observed in the data.[9]

4.4. Stock returns and investment returns

Following the Q-theory literature, from (24), we can define the return on the productive capital, $R_{K,t+1}$, as

$$R_{K,t+1} = \frac{1}{q_t}\left\{ q_{t+1} - \delta \mathbb{E}_{t+1}\left[\sum_{i=1}^{d+1} M_{t+1,t+i}\, u_{i-1}\, q_{t+i}\right] + \mathbb{E}_{t+1}\left[M_{t+1,t+d+1}\left(\alpha(1-\omega)K_{t+1}^{-\omega}W_{t+d}^{\omega}Z_{t+d+1}^{1-\alpha} \times \left[K_{t+1}^{1-\omega}W_{t+d}^{\omega}\right]^{\alpha-1} + q_{t+d+1}\frac{\partial g_{t+d+1}}{\partial K_{t+1}} - \frac{\partial h_{t+d+1}}{\partial K_{t+1}}\right)\right]\right\}. \tag{29}$$

Similarly, when there is inventory investment, the return on the inventory investment, $R_{W,t+1}$, is given by (21), as follows:

$$R_{W,t+1} = \frac{\alpha\omega Z_{t+1}^{1-\alpha}\left(K_{t-d+1}^{1-\omega}W_t^{\omega}\right)^{\alpha-1}K_{t-d+1}^{1-\omega}W_t^{\omega-1} + 1}{\frac{\partial h_t}{\partial W_t} + 1}. \tag{30}$$

Hence, the total investment returns, $R_{I,t+1}$, can be defined as the value-weighted average returns on the productive capital and inventory investment if inventory is present:

$$R_{I,t+1} = \frac{R_{K,t+1}K_{t+1} + R_{W,t+1}W_t}{K_{t+1} + W_t}, \tag{31}$$

which satisfies $\mathbb{E}_t[M_{t,t+1}R_{I,t+1}] = 1$.

Q-theory implies that stock returns equal investment returns state-by-state when production function exhibits constant returns to scale. However, in the main model, output is decreasing returns to scale. Moreover, inventory, TTB, and TTP expand the state space. Thus, in this economy, marginal productivity and average productivity are usually different. As a result, stock returns and investment returns are usually different. In fact, stock returns are the value-weighted average of returns on the productive capital, returns on inventory investment, returns on investment of incomplete projects initiated previously due to the TTB constraint, and returns on incomplete production initiated previously due to the TTP constraint.

Table 4 compares stock returns and investment returns from various models to evaluate the impacts of inventory investment, TTB, and TTP constraints.[10] First, adding inventory into Benchmarks 2, 4, and 6 decreases the risk premium of investment returns and stock returns, as compared with Benchmarks 1, 3, and 5, respectively. Second, a TTB or TTP constraint increases the risk premium of investment returns and stock returns in Benchmarks 3 and 5, as compared with Benchmark 1. Third, we see that the investment returns are very close to the stock returns in Benchmark 1, even though the production function exhibits decreasing returns to scale. However, when inventory or TTB and TTP constraints are added to the models, investment returns are higher than the stock returns in Benchmarks

[9] However, this result cannot be interpreted as evidence to support the view that variation in the dividend-price ratio comes mainly from discount rates, since the model is not designed to address the debate on predictability.

[10] Quantitatively, as the return on inventory investment is very small and inventory is only 7.03% of capital stock in the data, the investment return is dominated by the return on productive capital.

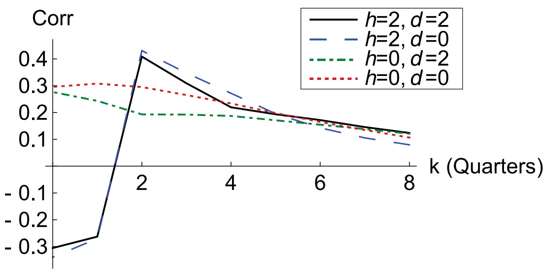


Fig. 4. Correlations between returns and investment growth rates. This figure depicts the cross-correlation between stock returns and investment growth rates, $Corr(R_t, \Delta I_{t+k})$, from the simulated quarterly data from four benchmark models and the main model ($h = 2, d = 2$). Benchmark models include Benchmark 2 ($h = 0, d = 0$), Benchmark 4 ($h = 2, d = 0$), and Benchmark 6 ($h = 0, d = 2$). See Table 2 for a detailed description of these models. The cross-correlation is computed in each sample path and the average over the 1,000 sample paths is reported.

2–6, while investment returns are lower than the stock returns in the main model. This signals high nonlinearity in these models when the decreasing returns to scale production function is combined with these features. Last, investment returns are less volatile than stock returns in all models except Benchmark 3, in which investment return volatility is close to the stock return volatility.[11] Overall, the main model shows that investment returns account for 79% of stock returns while contributing 93% to the volatilities.

Q-theory suggests a perfectly positive correlation between stock returns and investment returns, which also implies a positive contemporaneous correlation between stock returns and investment growth. However, Liu, Whited, and Zhang (2009) find a significantly negative contemporaneous correlation around −0.2, together with a significantly positive correlation between lagged stock returns and investment growth rate. Liu, Whited, and Zhang (2009) conjecture investment lags could contribute to this finding. Fig. 4 plots the cross-correlation between stock returns and investment growth from four different models. We see that stock returns and investment growth move simultaneously in the models without a TTB constraint (Benchmarks 2 and 6); i.e., there is a significantly positive contemporaneous correlation at $k = 0$. When TTB ($h = 2$) is present, Benchmark 4 and the main model show that stock returns and investment growth have a significantly negative contemporaneous correlation at $k = 0$ and a significantly positive correlation at $k = 2$. This confirms the conjecture in Liu, Whited, and Zhang (2009). That is, the TTB constraint drives their empirical findings.

4.5. Alternative specifications

4.5.1. TTB and consumption volatility

Table 2 shows that inventory is necessary to smooth extremely volatile consumption generated by the TTB constraint when the investment expenditures of a project occur mostly in the later periods. Table 5 further explores the TTB-only model with different investment expenditure schemes. In addition to Benchmark 3 in which investment expenditures occur mostly in the later periods (i.e., $w = \{0.1, 0.1, 0.8\}$), Table 5 also presents a case when a new project is fully invested in the first period (i.e., $w = \{1, 0, 0\}$), and another case when the investment of a new project is equally spread over three periods (i.e., $w = \{0.34, 0.33, 0.33\}$), which is similar to the case of Kydland and Prescott (1982).

The main impact of different investment expenditure schemes can be seen from investment and consumption volatilities. Investment becomes much riskier when most investment expenditures happen in the later periods. As a result, firms do not vary investment a lot when facing technology shocks, which makes investment very stable and consumption very volatile. For example, when $w = \{1, 0, 0\}$, consumption has a volatility of 0.63%, compared with 1.66% when $w = \{0.34, 0.33, 0.33\}$ and 1.76% when $w = \{0.1, 0.1, 0.8\}$. Correspondingly, the risk premium is much smaller in the models in which most investment expenditures happen in earlier periods, because payouts are countercyclical in these cases.[12] For example, results from the model with $w = \{1, 0, 0\}$ are very similar to those from the standard RBC model (Benchmark 1 in Table 2). Clearly, it is critical to have most investment expenditures occur in later periods to create procyclical payouts, which generates a sizable risk premium. However, such an investment expenditure scheme also results in a very volatile consumption stream. Thus, inventory is necessary in smoothing consumption in such cases.

4.5.2. Exploring the elasticity of intertemporal substitution

Previous calibrations use a small EIS. However, the long-run risk literature suggests a high EIS.[13] This subsection further explores the effects of the EIS on macroeconomic quantities and asset prices. Table 5 presents calibration results of an alternative model similar to the main model but with a high EIS value of 1.5.

Comparing the main model in Table 2 with the alternative model in Table 5, the output volatility does not vary a lot with the EIS, but the consumption volatility substantially increases with the EIS while investment and inventory become less volatile when the EIS is higher. The reason is that the propensity of smoothing consumption over time weakens when the EIS increases. Thus, the agent is willing to accept a more volatile consumption stream when her EIS is high. Given the fixed volatility of aggregate output, we see that the volatilities of investment and inventory decline with EIS. The volatilities of consumption and investment are far away from the data when the EIS is high. For instance, the consumption volatility is even higher than the output volatility when EIS = 1.5. Turning to the asset prices, we see that the price of risk decreases

[11] Empirically, Cochrane (1991) finds that aggregate investment return volatility is about 60% of stock market volatility. Liu, Whited, and Zhang (2009) also report that when matching only expected returns, the predicted investment return volatility is lower than stock return volatility.

[12] The unreported results on the impulse response of payouts confirm this.

[13] The empirical evidence on the aggregate EIS parameter size is mixed (see, e.g., Campbell, 2003, for a summary).

Table 5
Calibrations: Alternative specifications.
This table summarizes key moments of macroeconomic quantities and asset prices from calibrations of the TTB-only model and the alternative main model with different parameterizations. The TTB-only model is the standard RBC model with a 3-quarter ($h = 2$) time-to-build constraint. Three different cases of capital investment expenditure schemes are considered, i.e., $w = \{1, 0, 0\}$, $w = \{0.34, 0.33, 0.33\}$, and $w = \{0.1, 0.1, 0.8\}$. The alternative model is similar to the main model but with an elasticity of intertemporal substitution (EIS) of 1.5. All models include capital adjustment costs. The empirical data are from the NIPA tables and the annual Fama-French factors over 1964–2012. The numbers in parentheses are unlevered market returns, assuming a debt-to-equity ratio of 0.5 (see, e.g., Barro, 2006). The macroeconomic quantities are reported as quarterly, while the asset prices are annualized. The volatilities of output, consumption, and investment are computed from the Hodrick-Prescott filter. All moments are reported in percentages, except the Sharpe ratio.

	U.S. data	TTB only, No inventory ($h = 2$, $d = 0$)			Alternative main model
	(1964–2012)	$w = \{1, 0, 0\}$	$w = \{0.34, 0.33, 0.33\}$	$w = \{0.1, 0.1, 0.8\}$	Inventory ($h = 2$, $d = 2$), EIS = 1.5
Panel A: Macroeconomic quantities					
Volatility of output					
$\sigma(Y)$	1.55	1.42	1.46	1.46	1.55
Volatility of consumption					
$\sigma(C)$	0.84	0.63	1.66	1.76	1.69
Volatility of investment					
$\sigma(I)$	5.11	3.89	1.11	0.99	0.85
Mean and volatility of the inventory/consumption ratio					
W/C	30.03				26.41
$\sigma(W/C)$	7.92				0.77
Mean and volatility of the inventory/capital ratio					
W/K	7.03				7.13
$\sigma(W/K)$	1.65				0.45
Panel B: Asset prices					
Mean and volatility of the risk-free rate					
$E[R_f]$	1.57	3.78	5.17	1.52	5.23
$\sigma(R_f)$	2.18	1.50	6.62	17.81	0.32
Mean and volatility of the return to dividend claims					
$E[R_D]$	7.51 (5.53)	5.41	8.52	6.47	5.34
$\sigma(R_D)$	18.04 (12.03)	9.35	13.18	19.12	2.60
Risk premium of the return to dividend claims					
$E[R_D - R_f]$	5.94 (3.96)	1.63	3.35	4.95	0.11
Sharpe ratio of dividend claims					
$E[R_D - R_f]/\sigma(R_D)$	0.33	0.17	0.25	0.26	0.04
Mean and volatility of the return to consumption claims					
$E[R_C]$		5.74	9.51	11.64	5.35
$\sigma(R_C)$		12.06	12.74	26.64	1.95
Risk premium of the return to consumption claims					
$E[R_C - R_f]$		1.96	4.34	10.12	0.12
Sharpe ratio of consumption claims					
$E[R_C - R_f]/\sigma(R_C)$		0.16	0.34	0.38	0.06

with the EIS since the agent is less averse to the intertemporal substitution. Consequently, the risk premium and the Sharpe ratio drop significantly when the EIS is high. For example, the equity premium is only 0.11% when EIS=1.5. The overall evidence in Table 5 demonstrates that a small EIS is required to match both macroeconomic quantities and asset prices.

4.6. Applications

4.6.1. Asset prices and the business cycle

Empirically, asset prices tend to lead the business cycle. For example, Backus, Routledge, and Zin (2007, 2010) find robust evidence that financial variables, including equity returns, bond yields, and commodities, lead macroeconomic quantities by roughly two quarters. These cross-correlations are at odds with the standard RBC models in which everything moves simultaneously. Now I ask whether delays in production can generate the lead-lag patterns observed in the data, and more importantly, which feature contributes to these patterns.

Panel (a) of Fig. 5 replicates (Backus, Routledge, and Zin, 2010), plotting the cross-correlation between excess market returns and industrial production growth over 1964–2012, using monthly data. The largest and most significant correlation, 0.25, appears at $k = 7$ in Panel (a). That is, asset prices lead macroeconomic quantities by roughly two quarters.

Panel (b) depicts cross-correlations between excess stock returns and output growth based on the simulated quarterly data from various models. When there is no TTB constraint (Benchmarks 2 and 6), the largest correlation appears at $k = 0$; i.e., stock returns and output growth move simultaneously, as in a standard RBC model. When TTB is introduced, Panel (b) shows that the largest and most significant correlation appears at $k = 2$, which implies that stock returns lead output growth by two quarters. Examining Panel (b), we see that the main model generates a correlation of 0.34 , whereas Benchmark 4 generates a correlation of 0.44, which is too large when compared with the data. Clearly, the length of TTB determines the length of lags between returns and

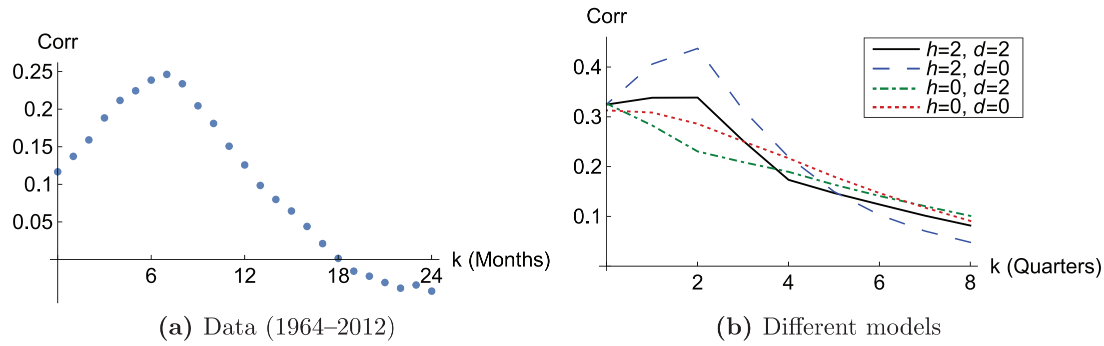


Fig. 5. Cross-correlations between excess returns and output growth rates. Panel (a) plots the cross-correlation between excess market returns and industrial production growth rates, $Corr(R_t - R_{f,t}, \Delta Y_{t+k})$, over 1964–2012, using monthly data. Panel (b) depicts the cross-correlation between excess stock returns and output growth rates from the simulated quarterly data from four benchmark models and the main model ($h = 2, d = 2$). Benchmark models include Benchmark 2 ($h = 0, d = 0$), Benchmark 4 ($h = 2, d = 0$), and Benchmark 6 ($h = 0, d = 2$). See Table 2 for a detailed description of these models. The cross-correlation is computed in each sample path, and the average over the 1,000 sample paths is reported.

Table 6
Investment regressions.
This table compares investment regressions from the empirical data and the main model. The investment-capital ratio is regressed against q, cash flow-capital ratio, and the lagged investment-capital ratio. The empirical results are from Eberly, Rebelo, and Vincent (2012) (Table 2). For the main model, the median values of regressions over 1,000 sample paths are reported. The Newey-West standard errors with two lags are reported in parentheses.

Regression	1		2		3	
	Empirical	Model	Empirical	Model	Empirical	Model
$Ln(Q_t)$	0.0331	0.0422			0.0126	0.0179
	(0.0023)	(0.0018)			(0.0019)	(0.0015)
$Ln(Cash\ Flow_t/K_t)$	0.0387	−0.0036			0.0170	0.0002
	(0.0024)	(0.0019)			(0.0020)	(0.0010)
I_{t-1}/K_{t-1}			0.7515	0.9355	0.6253	0.6140
			(0.0116)	(0.0254)	(0.0132)	(0.0317)
R^2	0.34	0.81	0.57	0.88	0.61	0.92

output growth, while the TTP feature helps to match the magnitude of correlation.

4.6.2. Investment regressions

Investment regressions find that lagged investment is a stronger predictor of investment than Tobin's Q and cash flows (e.g., Eberly, Rebelo, and Vincent, 2012). This lagged investment effect challenges the Q-based investment literature . Eberly, Rebelo, and Vincent (2012) use an investment adjustment cost model to generate such a lagged investment effect. Since the investment adjustment costs are specified as a convex function of investment growth rate, current investments are linked to the lagged investments. The TTB constraint considered in this paper provides a similar explanation to the lagged investment effect. Since current total investments include capital expenditures of incomplete projects initiated several periods ago, total investments are serially correlated. Table 6 compares investment regressions from the data and the main model. Overall, the main model generates results reasonably close to the data. For example, the main model exhibits a strong lagged investment effect, and Tobin's Q is also a good predictor of investments.[14] The main model does not generate the cash flow effect.[15]

5. Conclusions

This paper studies equilibrium asset prices and macroeconomic quantities in a dynamic production-based equilibrium model with production delays and inventory. These production delays naturally introduce risks into the economy. From a macroeconomic perspective, inventory is necessary to smooth the excessive consumption volatility generated by the TTB constraint when most investment expenditures happen in the later periods. TTP is necessary to capture substantial inventory holdings observed in the data. Moreover, the TTB constraint helps generate procyclical payouts, while TTP amplifies such procyclicality. From an asset pricing perspective, such procyclical payouts create a sizable equity premium. As inventory is less risky than the capital investment, it helps generate a less

[14] Untabulated results show that the lagged investment effect is introduced by the TTB feature, but not the TTP feature.

[15] Eberly, Rebelo, and Vincent (2012) find that the cash flow effect is not robust in the data.

volatile risk-free rate and a moderate term premium even when EIS is small. Quantitatively, this model generates equity returns close to the unlevered stock returns observed in the data and produces the lead-lag patterns between asset prices and macroeconomic quantities, the lagged investment effect, and the return predictability observed in the data. Both TTB and TTP constraints and inventory are necessary to match all these dimensions.

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