

Risk Aversion Sensitive Real Business Cycles

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Abstract. Technology choice allows for substitution of production across states of nature and depends on state-dependent risk aversion. In equilibrium, endogenous technology choice can counter a persistent negative productivity shock with an increase in investment. An increase in risk aversion intensifies transformation across states, which directly leads to higher investment volatility. In our model and the data, the conditional volatility of investment correlates negatively with the price-dividend ratio and predicts excess stock market returns. In addition, the same mechanism generates predictability of consumption growth and produces fluctuations in the risk-free rate.

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1. Introduction

In the standard real business cycle (RBC) model, production plans are made one period ahead implying that current period capital is fixed across states of nature. Thus, only exogenous shocks but no endogenous current period choices within the representative firm drive output across states of nature. In such an economic environment, the output risk is completely exogenous and independent of the firm's technology or the representative agent's preferences through risk aversion. Further, risk aversion has only small second-order effects on the dynamics of the macroeconomy, as shown in Tallarini (2000).¹ Any (time-series) variations in risk aversion should have unnoticeable effects on quantities such as consumption or investment. Yet, cyclical variations in risk aversion play a prominent role in explaining the variations in expected excess returns of the stock market in many theoretical works within the consumption-based asset pricing framework, in which consumption is exogenous.² In this paper, we show that with a more plausible state-dependent production technology, cyclical variations in risk aversion can jointly drive variations in asset prices and the macroeconomy. In our model, the conditional volatility of investment growth evolves procyclically relative to risk aversion, correlates negatively with the price-dividend ratio of the stock market, and predicts excess stock market returns. Consistent with the model, we see in the data a negative correlation between the conditional volatility

of investment and the price-dividend ratio of the stock market and that the conditional volatility of investment predicts (excess) stock market returns.

Ideally, we would like to provide microfoundations for the stylized production technology employed in the model, which we borrow from Cochrane (1993). Although we do provide a sketch for how an aggregate state-dependent production technology can emerge from aggregation of technologies per good and then aggregation of goods to total output, this, however, is only one step in that direction.³ Instead, we entertain the hypothesis that if technology choice and variations in risk aversion drive asset prices and macroeconomic quantities as in our model, then we should find empirical evidence for such a relation. Specifically, in our model, the state-dependent endogenous productivity is chosen optimally one period ahead, conditional on the exogenous time-varying risk aversion and the exogenous persistent productivity risk.⁴ In equilibrium, technology choice and time-varying risk aversion induce the conditional volatility of investments to vary. In contrast, otherwise comparable production-based asset pricing models, such as Kaltenbrunner and Lochstoer (2010), typically do not produce variation in the conditional volatility of investment and, hence, cannot speak to our empirical finding that the estimated conditional volatility of investment growth is countercyclical.

Conditional on risk aversion, technology choice allows modifying the risk of the total factor productivity growth.

Hence, one could conjecture that an increase in risk aversion decreases the volatility of output and investment. However, in the data, the volatility of the growth rate of output and investment increases in recessions. In our preferred specification of the model, the endogenous technology choice moves counter to a persistent productivity shock. This implies that investment declines less than in an economy without technology choice or that it even increases when facing a negative productivity shock. As shocks are persistent in the model, this can imply that investment first moves counter to a negative exogenous shock and only declines with a lag relative to investment in a benchmark economy. With an increase in risk aversion, investment reacts even more positively to a negative exogenous shock. Thereby, with endogenous technology choice, an increase in risk aversion leads to an increase in the volatility of investment.

Inspecting the log-linear solution of our model, we see that with technology choice risk aversion affects the conditional volatilities of macroeconomic variables. Specifically, when risk aversion is time varying, then the conditional volatility of investment evolves with risk aversion. When risk aversion is constant in a variant of our model with technology choice, then the conditional volatility of investment is also constant. When there is no technology choice and no variations in risk aversion, then the model collapses to the model of Kaltenbrunner and Lochstoer (2010) with recursive preferences and an exogenous productivity that follows an AR(1) process. Calibrating the models, we see that they perform equally well on the chosen macroeconomic quantities and they all match the Sharpe ratio of the stock market.

Technology choice is governed by a parameter, which determines how costly it is to transform productivity. The parameter is pinned down by calibrating the model to the volatility of the risk-free rate, which the model without technology cannot match. Because in our preferred calibration technology choice and risk aversion move counter to an exogenous shock, it delays the reaction not only of investment but also of consumption to a shock. As a result, technology choice generates predictability in consumption growth. The predictability generates fluctuations in the risk-free rate; but because it is short lived, it does not affect the dividend and consumption claims. Consequently, technology choice increases the volatility of the risk-free rate and reduces the correlation of the risk-free rate with the price-dividend ratio, making it statistically indistinguishable from the correlation in the data.

Regressing excess stock market returns on the log price-dividend ratio, we see that the models with time-varying risk aversion and with and without technology choice produce predictability that is statistically indistinguishable from the data. Further, without targeting

it, our model with technology choice and time-varying risk aversion reproduces the variations in the conditional volatility of investment and its correlation with the log price-dividend ratio of the stock market, but only with a high elasticity of intertemporal substitution (EIS). Finally, when regressing excess stock market returns on the conditional volatility of investment growth, we see that the model with time-varying risk aversion, technology choice, and high EIS produces predictability that is statistically indistinguishable from the data.

Our paper speaks to the literature that explores the asset pricing implications of production transformation across states or technologies. To allow for production transformation across states, Cochrane (1993) proposes to allow firms to choose state-contingent productivity endogenously subject to a constraint set. In closely related works, Cochrane (1988) and Jermann (2010) back out the stochastic discount factor from producers' first-order conditions assuming complete technologies, that is, that there are as many technologies as states of nature. In similar spirit, Belo (2010) applies state-contingent productivity to derive a pure production-based pricing kernel in a partial equilibrium setting, which gives rise to a macrofactor asset pricing model that explains the cross-sectional variation in average stock returns. The takeaway from these papers is that state-contingent technology can explain asset prices in both the time-series and the cross section and that the way the economy substitutes productivity across states is related to asset prices, suggesting that risk aversion matters for the macroeconomy. However, these studies do not look at the joint implications of state-contingent technology for asset prices and the macroeconomy. Our paper fills this gap in the literature.⁵

Seminal contributions to the literature on investment- or production-based asset pricing include Jermann (1998) who introduces habit formation and capital adjustment costs and Boldrin et al. (2001) who introduce, in addition, two sectors in the standard RBC model to explain the equity premium and the stock return volatility. Kaltenbrunner and Lochstoer (2010) introduce Epstein-Zin preferences in the standard RBC model with capital adjustment costs, in which the persistence in capital generates long-run risk à la Bansal and Yaron (2004). Kaltenbrunner and Lochstoer (2010) explain the stock market Sharpe ratio, with high EIS, or also the stock market equity premium and stock return volatility, with low EIS. With high EIS, they also explain the stock market return volatility for a dividend claim that resembles the stock market dividends.⁶ We build on Kaltenbrunner and Lochstoer (2010) by introducing technology choice and time-varying risk aversion. Time-varying risk aversion generates excess return predictability and, through that, explains the

volatility of the price-dividend ratio of the dividend claim. With technology choice, the macroeconomy reacts to changes in risk aversion by varying the volatility of output, investment, and consumption and, through that, links the macroeconomy to asset prices.

2. A Macrofinance Economy with State-Contingent Technology

Consider a representative agent who owns an all-equity representative firm, which uses productive capital to generate one real good and operates in discrete time with infinite horizon.

2.1. The Firm

Output, Y_t , is given by

$$Y_t = K_t^\alpha \Omega_t^{1-\alpha}, \quad (1)$$

where K_t denotes the capital stock at the beginning of period t and Ω_t is total factor productivity. The constant parameter $\alpha \in (0, 1)$ stands for the capital share in output.

Capital accumulates according to

$$K_{t+1} = (1 - \delta)K_t + g_t, \quad (2)$$

where δ is the depreciation rate and g_t stands for the capital formation function. We specify g as in Jermann (1998), that is,

$$g_t = \left[\frac{a_1}{1 - 1/\chi} \left(\frac{I_t}{K_t} \right)^{1-1/\chi} + a_2 \right] K_t, \quad (3)$$

where I_t denotes investment at time t , the curvature $\chi > 0$ governs capital adjustment costs, and a_1 and a_2 are constants. These specifications imply that capital adjustment costs are high when χ is low and that capital adjustments are costless when $\chi \rightarrow \infty$. Following Boldrin et al. (2001), we set a_1 and a_2 such that there is no cost to capital adjustment in the deterministic steady-state

$$a_1 = (e^\mu - 1 + \delta)^{1/\chi} \quad \text{and} \quad a_2 = \frac{1}{1 - \chi} (e^\mu - 1 + \delta),$$

where μ is the average growth rate of the economy.

2.2. Productivity and Technology Choice

Departing from the standard business cycle setting, we assume that the representative firm modifies the underlying natural productivity Θ_t . Following Cochrane (1993) and Belo (2010), at time t , a state-contingent technology (or measured total factor productivity) $\Omega_{t+1} = \Omega(t, \Theta_{t+1})$ is chosen, that is, the representative firm chooses total factor productivity (TFP) as a function of the exogenous state of the economy, through a Constant Elasticity of Substitution aggregator

$$\mathbb{E}_t \left[\frac{\Omega_{t+1}^{(1-\alpha)\nu}}{\Theta_{t+1}^{(1-\alpha)\nu}} \right] \leq 1, \quad (4)$$

where \mathbb{E}_t is the conditional expectation operator and where $\log \Theta_t$ follows an AR(1) process with trend,

$$\log \Theta_{t+1} = \log Z_{t+1} + \phi(\log \Theta_t - \log Z_t) + \epsilon_{t+1}, \quad \text{and} \\ \log Z_t = \mu t, \quad (5)$$

with $|\phi| < 1$ and ϵ_t is the independent and identically distributed (i.i.d.) $N(0, \sigma^2)$ exogenous shock.

In (4), the curvature ν captures the representative firm's technical ability to modify technology. When $\nu < 1$, increasing the volatility of Ω_{t+1} also increases average productivity. For this reason, we assume that $\nu > 1$. With this assumption, as ν increases, distorting the underlying shocks reduces average productivity. When $\nu \rightarrow +\infty$, it is infinitely costly to modify the exogenous productivity. Therefore, we obtain $\Omega_{t+1} = \Theta_{t+1}$.

Online Appendix A provides intuition for the reduced-form approach in modeling technology choice. Briefly, one way to consider the reduced-form specification of technology choice is that it represents the ability of the economy to allocate productive capacity across states, through choosing the mixture of different technologies. The mean-variance characteristics of aggregate productivity depend on the allocation of the aggregate capital across technologies, where higher risk leads to higher average productivity. Thus, we interpret the technology modifications set in (4) as a simple abstract form of modeling state-contingent technologies implying flexibility for optimal future productivity. More specifically, constraint (4) determines the representative firm's ability to trade off higher realizations of shocks in some states at time $t + 1$ with lower realizations in other states. The optimal choice offsets the marginal benefit from smoothing consumption over time and states with the marginal cost of lower average productivity (or a trade-off between static efficiency and flexibility similar to Mills and Schumann 1985).

Another way of viewing technology choice is that it represents the ability of the representative agent to control the aggregate productivity risk, through employing certain resources. The costs in employing such resources naturally reduce productivity risk and average productivity. The wedge between Ω and Θ is, then, simply the difference between the ex post productivity after employing the resources to control risk and the benchmark case where no controls were employed. A positive wedge then represents the case where the controls turned out to be beneficial, because they resulted in higher productivity, whereas a negative wedge represents the case where ex post the controls resulted in lower productivity. The representative agent then chooses how many resources to employ, depending on their effect on productivity risk and risk aversion. A large ν in this case implies that large

costs need to be incurred to reduce the aggregate productivity risk by a small amount.

2.3. The Household

To separate the EIS from risk aversion, we assume that the representative agent exhibits recursive preferences (Kreps and Porteus 1978; Epstein and Zin 1989, 1991; and Weil 1989), whose utility at time t is represented by

$$U_t = \left\{ (1 - \beta)C_t^{1-1/\psi} + \beta \mathbb{E}_t \left[U_{t+1}^{1-\gamma_t} \right]^{\frac{1-1/\psi}{1-\gamma_t}} \right\}^{\frac{1}{1-1/\psi}}, \quad (6)$$

where $0 < \beta < 1$ denotes the subjective time discount factor, C_t stands for aggregate consumption at time t , $\psi > 0$ represents the EIS, and γ denotes the state-dependent relative risk aversion.

2.4. State-Dependent Relative Risk Aversion

For parsimony and computational tractability, we assume that the state-dependent relative risk aversion γ_t depends only on the exogenous technological productivity level at time t . It is given by

$$\gamma_t = \gamma - (\eta_1 - \eta_2 \theta_t) \theta_t, \quad (7)$$

where γ is the steady-state level of risk aversion, η_1 and η_2 are constant coefficients, and $\theta_t = \log(\Theta_t/Z_t)$. This specification allows for joint cyclical variations in risk aversion and asset pricing moments, consistently with, for example, Campbell and Cochrane (1999), where risk aversion is endogenous and state dependent.

2.5. The Maximization Problem

Every period the representative agent maximizes his or her utility (6) by choosing consumption C_t , investment I_t , and the productivity Ω_{t+1} for every state next period. The maximization problem is expressed as follows:⁷

$$\begin{aligned} &U(Z_t, \Theta_t, K_t, \Omega_t) \\ &= \max_{C_t, I_t, \{\Omega(t, \Theta_{t+1})\}_{\Theta_{t+1} \in (0, \infty)}} \left\{ (1 - \beta)C_t^{1-1/\psi} + \beta \mathbb{E}_t \left[U_{t+1}^{1-\gamma_t} \right]^{\frac{1-1/\psi}{1-\gamma_t}} \right\}^{\frac{1}{1-1/\psi}} \\ &\text{s.t.} \quad Y_t = C_t + I_t, \end{aligned} \quad (8)$$

where, with slight abuse of notation, $U_t = U(Z_t, \Theta_t, K_t, \Omega_t)$ represents the maximized utility and (8) states the resource constraint or market clearing. The state variables K_t and Ω_t determine the level of output according to (1), K_t together with I_t determine capital next period according to (2) and (3), the exogenous productivity Θ_t determines the conditional distribution of Θ_{t+1} according to (5), and the conditional distribution of the exogenous productivity Θ_{t+1} together

with constraint (4) determine the trade-off in choosing the productivity Ω_{t+1} in every state next period.

2.6. Asset Prices

Besides the macroeconomic quantities, we also study asset prices. Specifically, we compute the returns $R_{f,t}$ on the risk-free asset, which pays one unit of consumption next period, and the returns $R_{i,t}$ on real investment, which are equal to the returns on the aggregate consumption claim (Restoy and Rockinger, 1994). In addition, we study the returns on a risky stock with next period dividends, D_{t+1} , as follows

$$R_{s,t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}, \quad (9)$$

where P_t denotes the price of the dividend claim at time t . Because the properties of the dividends of the representative firm generated by the model differ from those of the dividends of the aggregate stock market in the data, we also price a claim to a dividend process. Following Kaltenbrunner and Lochstoer (2010), we assume that the log growth in dividend, denoted by $\Delta d_{t+1} = \log(D_{t+1}/D_t)$, evolves according to

$$\Delta d_{t+1} = \mu + d_1(\theta_t - c_t) + d_2 \epsilon_{t+1} + d_3 \epsilon_{t+1}^d, \quad (10)$$

where c_t denotes log deviations of consumption, normalized by Z , from its steady state. Further, ϵ_t^d is i.i.d. $N(0, 1)$, uncorrelated with ϵ_t , and d_1, d_2, d_3 are constant coefficients.

2.7. The Equilibrium Conditions

The optimal amount of investment in period t is characterized by the marginal q condition,

$$\frac{1}{g_{I,t}} = \mathbb{E}_t \left[M_{t,t+1} \left(\alpha \frac{Y_{t+1}}{K_{t+1}} + \frac{1 - \delta + g_{K,t+1}}{g_{I,t+1}} \right) \right], \quad (11)$$

where $g_{I,t}$ and $g_{K,t}$ are the partial derivatives of the capital formation function (3) with respect to investment and capital, respectively, in period t . The stochastic discount factor, M^8 , which is given by

$$M_{t,t+1} = \beta \left[\frac{C_{t+1}}{C_t} \right]^{-\frac{1}{\psi}} \left[\frac{U_{t+1}^{1-\gamma_t}}{\mathbb{E}_t(U_{t+1}^{1-\gamma_t})} \right]^{\frac{1}{\psi} \gamma_t}. \quad (12)$$

The left-hand side of (11) shows the marginal cost of investment, which is the amount of investment required to generate a unit of productive capital. The right-hand side of (11) describes the marginal benefit from an additional unit of capital, which stems from next period's marginal product of capital and the remaining marginal value of future capital stock. Thus, the firm optimally equates the marginal costs

with the marginal benefits of investment. From this first-order condition, returns on an additional unit of investment are

$$R_{i,t+1} = g_{I,t} \left(\alpha \frac{Y_{t+1}}{K_{t+1}} + \frac{1 - \delta + g_{K,t+1}}{g_{I,t+1}} \right). \quad (13)$$

Finally, the representative firm in a period t optimally chooses the productivity Ω_{t+1} state by state for the next period, as follows

$$\left(\frac{\Omega_{t+1}}{\Theta_{t+1}} \right)^{(1-\alpha)v} = \frac{(M_{t,t+1} \Theta_{t+1}^{1-\alpha})^{\frac{v}{v-1}}}{\mathbb{E}_t \left[(M_{t,t+1} \Theta_{t+1}^{1-\alpha})^{\frac{v}{v-1}} \right]}, \quad (14)$$

where the ratio on the left-hand side is the transformation of the exogenous productivity. Equation (14) describes the tradeoff embedded in the distribution of Ω . On the one hand, it can be beneficial to increase productivity in states where the productivity is exogenously high and decrease it where the productivity is exogenously low. In this way, next period's average productivity is maximized because the cost of deviating from the exogenous productivity is a function of the ratio of transformation.⁹ We see this from the case of Constant Relative Risk Aversion (CRRA) preferences, $\gamma_t = 1/\psi$, and risk neutrality, $\gamma_t = 0$, where the stochastic discount factor is constant and, thus, cancels out from (14). As a result, the log optimal endogenous technology is proportional to the log exogenous productivity,

$$\log \Omega_{t+1} \propto \frac{v}{v-1} \log \Theta_{t+1}. \quad (15)$$

On the other hand, when the representative agent is risk averse, it is optimal to shift productivity to high “value” states, that is, states of high marginal utility M . Given the above tradeoff in the model with endogenous technology choice, it can be optimal to amplify or reduce exogenous volatility and it can be optimal to choose a positive or negative correlation between endogenous and exogenous productivity.

Summing up, seven equilibrium conditions ((1), (2), (5), (6), (8), (11), and (14)), where condition (4) is implied by (14)) determine the dynamics of the seven quantities (Θ , K , Ω , C , I , Y , and U) that describe the behavior of the macroeconomy, in addition to the stochastic discount factor given by (12), which prices real and financial assets. Additional details are in Online Appendix B.1.

3. The Log-Linearized Real Economy

To understand the economic mechanism behind technology choice, we derive the log-linear approximation of the macroeconomic dynamics. Asset prices are then solved using a projection method utilizing the log-linear dynamics of the state vector.

The proposition below summarizes the log-linear economy in equilibrium, where lower-case letters denote percentage deviations from steady-state values of detrended variables. That is, defining $\tilde{X}_t = X_t/Z_t$ for some variable $X \in \{\Theta, K, \Omega, Y, C, I, U\}$ then $x_t = \log(\tilde{X}_t/\tilde{X})$, where \tilde{X} refers to the steady-state value.

Proposition 1. *The state vector of the economy is $(k_t, \omega_t, \theta_t)$ and, thus, for a macroeconomic variable $x_t \in \{y_t, c_t, i_t, u_t\}$ we have*

$$x_t = x_k k_t + x_\omega \omega_t + x_\theta \theta_t. \quad (16)$$

All coefficients are independent of the technology choice parameter v and risk aversion γ . The law of motion of the state vector is given by

$$\begin{aligned} \theta_{t+1} &= \phi \theta_t + \epsilon_{\theta,t+1}, \\ k_{t+1} &= \frac{1 - \delta}{e^\mu} k_t + \left(1 - \frac{1 - \delta}{e^\mu} \right) i_t, \\ \omega_{t+1} &= \phi \theta_t + \sigma_\omega(\gamma_t) \epsilon_{\theta,t+1}. \end{aligned} \quad (17)$$

The technology choice is represented by the sensitivity of the endogenous productivity to exogenous shocks, which is given by

$$\sigma_\omega(\gamma_t) = \frac{(1 - \alpha)v - \frac{1}{\psi} c_\theta - \left(\gamma_t - \frac{1}{\psi} \right) u_\theta}{(1 - \alpha)(v - 1) + \frac{1}{\psi} c_\omega + \left(\gamma_t - \frac{1}{\psi} \right) u_\omega}. \quad (18)$$

Online Appendix B contains proofs, additional details of the log-linearization, and expressions for all coefficients and the steady states.

In Proposition 1, the sensitivities with respect to ω , that is, x_ω , represent sensitivities with respect to the (measured) total factor productivity, whether this is endogenous, as in the case of technology choice, or exogenous, as in the standard RBC model. In the case of technology choice, θ also affects the macroeconomy because it controls the expected productivity and the magnitude of the effects depend on the persistence of the exogenous shocks. If $\phi = 0$, then all sensitivities with respect to θ are zero, that is, $x_\theta = 0$ for all $x \in \{y, c, i, u\}$.

Proposition 1 still holds for the case where γ_t is also driven by own shocks, in which case risk aversion becomes a state variable. Even then, Equation (16) does not change, because the elasticities with respect to risk aversion (x_γ) are zero, as shown in Online Appendix B.

Our main prediction is that technology choice with time-varying risk aversion produces conditional volatilities of macroeconomic variables that are time varying. Specifically, models without technology choice or without time-varying risk aversion imply constant conditional volatilities for all macroeconomic variables. Substituting the laws of motion of ω_t and θ_t into the equilibrium relation (16), we obtain the following result.

Corollary 1. *With technology choice ($v < \infty$) and time-varying risk aversion, the conditional volatility of a macroeconomic variable $x_t \in \{y_t, c_t, i_t, u_t\}$ is time-varying:*

$$x_{t+1} = x_k k_{t+1} + \tilde{x}_\theta \theta_t + \sigma_x(\gamma_t) \epsilon_{t+1}, \tag{19}$$

where $\tilde{x}_\theta = \phi(x_\omega + x_\theta)$ and $\sigma_x(\gamma_t) = \sigma_\omega(\gamma_t)x_\omega + x_\theta$ for $x \in \{y, c, i, u\}$.

This corollary shows that all the conditional volatilities are driven by the conditional volatility of ω , which is given by $\sigma_\omega(\gamma_t)$. It defines optimal technology choice.

3.1. Technology Choice, $\sigma_\omega(\gamma_t)$

Technology choice allows to optimally choose productivity risk over one period through σ_ω and is given by the equilibrium condition (18).¹⁰ It depends on how costly it is to transform technology, governed by v , the sensitivity of the log stochastic discount factor to θ , given by $-\frac{1}{\psi}c_\theta - (\gamma_t - \frac{1}{\psi})u_\theta$, and the sensitivity of the log stochastic discount factor to ω , given by $-\frac{1}{\psi}c_\omega - (\gamma_t - \frac{1}{\psi})u_\omega$.¹¹ Thus, it depends on all parameters. Yet, we express it only as a function of γ because it is the only parameter that varies over time. We start by looking at limit properties of technology choice.

Corollary 2. *The limits of technology choice are*

$$\lim_{v \rightarrow \infty} \sigma_\omega(\gamma_t) = 1 \quad \text{and} \quad \lim_{\gamma_t \rightarrow \infty} \sigma_\omega(\gamma_t) = -\frac{u_\theta}{u_\omega}.$$

When $\sigma_\omega(\gamma_t)$ equals unity, there is no transformation in productivity and we recover the standard RBC model. When $\sigma_\omega(\gamma_t) > 1$, it is optimal to choose amplified shocks that comove with the underlying shocks, that is, it is optimal to shift productivity from low-productivity states to high-productivity states. When $0 \leq \sigma_\omega(\gamma_t) \leq 1$, it is optimal to choose less volatile shocks that comove with the underlying shocks; when $-1 \leq \sigma_\omega(\gamma_t) \leq 0$, it is optimal to choose less volatile shocks that move counter to the underlying shocks. It is even possible to have $\sigma_\omega(\gamma_t) \leq -1$, in which case technology choice not only more than offsets the underlying shocks but also amplifies them.

In the model, the representative firm shifts productivity across states depending on the trade-off between maximizing average productivity and transferring productivity from low-value states to high-value states. On the one hand, the firm maximizes productivity by shifting it to states with high exogenous productivity, where the transformation cost is lower. This mechanism is driven by the terms $(1 - \alpha)v$ and $(1 - \alpha)(v - 1)$ in (18). When agents have risk-neutral CRRA utility ($\gamma = 1/\psi = 0$), then $\sigma_\omega = v/(v - 1)$, as the exact solution in (15). For this case, a lower v implies lower transformation cost and, thus, more productivity is shifted to high-productivity states.

On the other hand, when agents are risk averse, the firm also wants to shift productivity to high-value states. With $\gamma_t > 1/\psi$, the value of a state decreases with consumption and the continuation utility, as shown in (12). Consequently, technology choice offsets some of the fluctuations in consumption and continuation utility coming from exogenous shocks and $\sigma_\omega(\gamma_t)$ decreases with c_θ and u_θ . Naturally, the more sensitive is consumption and utility to exogenous shocks (higher c_θ and u_θ), the larger is the optimal shift in productivity. This can be seen from the numerator of $\sigma_\omega(\gamma_t)$. As productivity shifts to low-value states, consumption and the continuation utility increase in those states along with their value. Depending on the sensitivity of the value of a state to ω_t , which is determined by c_ω and u_ω , optimal technology choice pushes $\sigma_\omega(\gamma_t)$ toward zero, as shown by the denominator in (18). If $\sigma_\omega(\gamma_t) = 0$, then all one-period productivity risk is eliminated.

Regarding the cost of productivity transformation, we emphasize that the effects of c_θ , u_θ , c_ω , and u_ω depend on the cost of transformation, v . When v is high, less productivity is shifted through technology choice and $\sigma_\omega(\gamma_t)$ is close to one. In the limit ($v \rightarrow \infty$), we recover the standard RBC case with no shift in productivity.

3.2. Two Implications

The technology choice model generates two implications that directly link the macroeconomy with asset prices. According to the model, the cost of technology choice (v) controls the volatility of the risk-free rate. To see why, consider the expected growth rate in productivity, which is given by

$$\mathbb{E}_t(\log \Omega_{t+1} - \log \Omega_t) = \mu - \phi(1 - \phi)\theta_{t-1} + [\phi - \sigma_\omega(\gamma_{t-1})]\epsilon_t.$$

When the expected growth rate in productivity and, hence, output is high (low), the value of intertemporal substitution is high (low), which is reflected in a high (low) risk-free rate. More importantly, the larger is the fluctuations in the above expected growth rate the higher is the volatility of the risk-free rate, where the extent by which it fluctuates is determined by technology choice. Specifically, a decrease in the cost of technology choice (lower v) amplifies technology choice and pushes $\sigma_\omega(\gamma_t)$ further away from one, which can be inferred from

$$\frac{\partial \sigma_\omega(\gamma_t)}{\partial v} = \frac{(1 - \alpha)[1 - \sigma_\omega(\gamma_t)]}{(1 - \alpha)(v - 1) + \frac{1}{\psi}c_\omega + (\gamma_t - \frac{1}{\psi})u_\omega}.$$

As a result, the lower is v the higher is the risk-free rate volatility. In the calibration, we use this property of the model to pin down the value of v .

The second implication stems from the fact that c_θ and u_θ are nonzero if $\phi > 0$. That is, the value of a state is not only determined by the (measured) total factor productivity but also by the level of θ , because it determines expected future endogenous productivity. For this reason, even when risk aversion is infinite, the optimal technology choice does not eliminate all productivity risk but results in $\sigma_\omega(\gamma_t)$ being equal to $-u_\theta/u_\omega$, as shown in Corollary 2. All one-period risk is eliminated only when $\phi = 0$, in which case $u_\theta = 0$, and risk aversion is infinite.¹² Otherwise, it is optimal to more than offset exogenous productivity shocks, that is, $\sigma_\omega(\gamma_t)$ is negative. This is optimal when negative productivity shocks are very costly, because it allows building up capital as a response to such negative and persistent shocks. This can be seen from the fact that investment typically reacts negatively to θ , as can be inferred from $i_\theta = -c_\theta C/L$.¹³

The second implication, that is, whether $\sigma_\omega(\gamma_t)$ is positive or negative, determines how γ_t affects the conditional volatilities of output and investment. From Corollary 1, we know that the conditional volatility of investment is given by the absolute value of $\sigma_i(\gamma_t) = \sigma_\omega(\gamma_t)i_\omega + i_\theta$, where i_θ is typically negative; and the conditional volatility of output is given by the absolute value of $\sigma_y(\gamma_t) = \sigma_\omega(\gamma_t)y_\omega$, because $y_\theta = 0$. Further, the effect of risk aversion on technology choice is given by

$$\frac{\partial \sigma_\omega(\gamma_t)}{\partial \gamma_t} = -\frac{u_\theta + \sigma_\omega(\gamma_t)u_\omega}{(1-\alpha)(v-1) + \frac{1}{\psi}c_\omega + (\gamma_t - \frac{1}{\psi})u_\omega}.$$

The above expression is typically negative and not very sensitive to $\sigma_\omega(\gamma_t)$ because u_ω is much smaller than u_θ . As a result of the above properties, when $\sigma_\omega(\gamma_t)$ is positive, an increase in γ_t leads to a decrease in the conditional volatility of output. In this case, the conditional volatility of investment may decrease or increase depending on the relative magnitudes of i_ω and i_θ .

When $\sigma_\omega(\gamma_t)$ is negative, an increase in risk aversion leads to an increase in both output and investment volatility. In our calibrated model, $\sigma_\omega(\gamma_t)$ is on average negative, which generates a testable prediction, namely, that it makes the conditional volatilities of investment and output countercyclical.

3.3. Equivalence with Standard RBC

Our model predicts that the conditional volatility of TFP is stochastic and driven by risk aversion. However, because technology choice is not observable to an econometrician, a standard RBC model is equivalent to our technology choice model if the exogenous TFP process follows the same process as ω .

Corollary 3. *The log-linearized economy without technology choice ($v = \infty$), where the total factor productivity is given by*

$$\omega_{t+1} = \phi \theta_t + \tau_{\omega,t} \epsilon_{t+1},$$

is isomorphic in its pricing and macroeconomic implications with the log-linearized technology choice economy provided that $\tau_{\omega,t} = \sigma_\omega(\gamma_t)$.

The above result, together with Proposition 1 and Corollary 1, justifies modeling the macroeconomy with exogenously specified stochastic volatilities and shows how to model them to be consistent with technology choice.

Most importantly, however, our model predicts that the conditional volatilities are driven by risk aversion, which links the macroeconomy to stock returns, as we see later. What we will show is that, if conditional volatilities are exogenously driven by independent shocks, then this link breaks down in our setting.

4. Solution Method and Asset Prices

We solve the model numerically first by log-linearizing the economy. The risk-free rate is then obtained in closed form. The price-to-dividend ratios of the consumption claim and the dividend claim are solved numerically using a projection method. The relevant Euler condition for the dividend claim is given by

$$\frac{P_t}{D_t} = \mathbb{E}_t \left[M_{t,t+1} e^{\Delta d_{t+1}} \left(1 + \frac{P_{t+1}}{D_{t+1}} \right) \right], \quad (20)$$

and similarly for the consumption claim.

Starting with cash flows, the following proposition presents the log consumption growth for the log-linearized approximation of the model's equilibrium.

Proposition 2. *Given the log-linear approximation of the equilibrium in Proposition 1, the log consumption growth is conditionally normal, $\Delta c_{t+1} = \mu_t + \sigma_c(\gamma_t) \epsilon_{t+1}$. Its conditional mean is given by*

$$\mu_t = \mu + \mu_k k_t + \mu_\theta \theta_{t-1} + \sigma_\mu(\gamma_t) \epsilon_t, \quad (21)$$

where $\mu_k = \delta c_k(i_k - 1)$, $\mu_\theta = \delta c_k i_\theta - c_\theta(1 - \phi)$, and $\sigma_\mu(\gamma_t) = \delta c_k \sigma_i(\gamma_t) + c_\theta - \sigma_c(\gamma_t)$. The coefficients $\sigma_c(\gamma_t)$ and $\sigma_i(\gamma_t)$ are defined in Corollary 1.

The stochastic discount factor in (12) is also log-normally distributed in the log-linear approximation.

Proposition 3. *Given the log-linear approximation of equilibrium in Proposition 1, the log stochastic discount factor is conditionally normal:*

$$\log M_{t,t+1} = \log \hat{\beta} - \frac{1}{\psi} \mu_t - \sigma_m(\gamma_t) \epsilon_{t+1}, \quad (22)$$

where

$$\log \hat{\beta}(\gamma_t) = \log \beta + \frac{1}{2}(1 - \gamma_t) \left(\gamma_t - \frac{1}{\psi} \right) \sigma_u(\gamma_t)^2 \sigma^2, \quad (22a)$$

$$\sigma_m(\gamma_t) = \frac{1}{\psi} \sigma_c(\gamma_t) + \left(\gamma_t - \frac{1}{\psi} \right) \sigma_u(\gamma_t), \quad (22b)$$

where μ_t is given in Proposition 2, $\sigma_u(\gamma_t)$ is defined in Corollary 1, and σ denotes the standard deviation of the exogenous shock ϵ defined in (5).

The price of risk is given by the absolute value of $\sigma_m(\gamma_t)\sigma$. The introduction of technology choice changes the sensitivities to the exogenous shock, the σ_x 's, which in turn affects the price of risk. To see the effect of technology choice on the price of risk, we express $\sigma_m(\gamma_t)$ in terms of ν , that is, by substituting in the expressions of $\sigma_c(\gamma_t)$ and $\sigma_u(\gamma_t)$ as derived in Proposition 1:

$$\begin{aligned} \sigma_m(\gamma_t) &= \frac{(\nu - 1)A_\theta(\gamma_t) + \nu A_\omega(\gamma_t)}{\nu - 1 + A_\omega(\gamma_t)/(1 - \alpha)}, \quad (23) \\ A_x(\gamma_t) &= \frac{1}{\psi} c_x + \left(\gamma_t - \frac{1}{\psi} \right) u_x, \quad x \in \{\theta, \omega\}. \end{aligned}$$

Taking the first derivative with respect to ν gives

$$\frac{\partial \sigma_m(\gamma_t)}{\partial \nu} \propto \frac{A_\theta(\gamma_t) + A_\omega(\gamma_t)}{1 - \alpha} - 1, \quad (24)$$

which for reasonable parameters is positive. This implies that the more flexible is the technology choice (the lower is ν), the lower is the price of risk. In fact, at the lowest possible value ($\nu = 1$), we obtain $\sigma_m(\gamma_t) = 1 - \alpha$ in which case the price of risk becomes very low and risk aversion does not affect it.

Despite the effect of technology choice on the price of risk, the model is on the same footing as the standard RBC model in fitting the price of risk. The reason is as follows: The unconditional price of risk at the steady state is roughly determined by σ_c and σ_u . The coefficient σ_c is pinned down by the unconditional volatility of consumption growth, and the technology choice model can fit it. The coefficient σ_u is determined by the long-run volatility of μ_t .¹⁴ From Proposition 2 it follows that technology choice affects only σ_μ , but this coefficient has a small effect on the long-run volatility of μ_t , unless ν is close to one. As a result, both the technology choice and the standard RBC model can match the price of risk.

The following proposition provides the expression for the risk-free rate and an approximate expression of the log price-dividend ratio of the dividend claim.

Proposition 4. *Given the stochastic discount factor in Proposition 3, the continuously compounded one-period risk-free rate is*

$$r_{f,t} = -\log \hat{\beta}(\gamma_t) + \frac{1}{\psi} \mu_t - \frac{1}{2} \sigma_m(\gamma_t)^2 \sigma^2. \quad (25)$$

The log-linear approximation of the stock price-dividend ratio, assuming an AR(1) process for γ_t , is given by

$$p_t - d_t \approx \overline{p - d} + \mathbb{E}_t \sum_{\tau=0}^{\infty} \hat{\gamma}^\tau \xi_{t+\tau}, \quad (26)$$

where ξ_t is a zero mean variable given by

$$\begin{aligned} \xi_t &= d_1(\theta_t - c_t) - \frac{1}{\psi} (\mu_t - \mu) \\ &+ \left[\xi_1(\gamma)(\gamma_t - \gamma) + \frac{1}{2} \xi_2(\gamma)(\gamma_t - \gamma)^2 \right] \sigma^2. \quad (27) \end{aligned}$$

All expressions above are defined in Online Appendix C.

The level of the risk-free rate is principally determined by the subjective discount factor β and the EIS ψ . For standard parameters, the risk-free rate is driven mainly by fluctuations in expected consumption growth μ_t . That is, fluctuations in risk aversion are typically not quantitatively important for the volatility of the risk-free rate. What matters is technology choice. For reasonable parameters, $\sigma_\mu(\gamma_t)$ decreases with ν , which implies that a more flexible technology choice makes μ_t more volatile.

A fundamental asset pricing quantity is the volatility of the price-dividend ratio of the stock market. Proposition 4 helps us understand how the log price-dividend ratio of the dividend claim varies with the state of the economy. We see that three factors drive the $p - d$ ratio: (i) the expected consumption growth μ_t , which drives the risk-free rate, (ii) the expected cash-flow growth driven by $(\theta_t - c_t)$, and (iii) risk aversion. The first factor, namely, μ_t , is important only when intertemporal substitution is quite inelastic (i.e., when ψ is very low). Further, only the persistent fluctuations of μ_t coming from k_t and θ_{t-1} , in (21), matter because the fluctuations coming from ϵ_t are short lived. For this reason, technology choice has negligible effect on the volatility of the $p - d$ ratio.

The dividend growth driven by $(\theta_t - c_t)$ is potentially important for the fluctuations of the log price-dividend ratio if d_1 is large enough. Effectively, a positive shock to exogenous productivity that leads to a significant and persistent shock to dividend growth also leads to a substantial increase in the stock price relative to dividends.

Finally, the introduction of time variation in risk aversion can augment the stock market volatility, principally through the linear term $\xi_1(\gamma)(\gamma_t - \gamma)$. In consumption-based asset pricing models, for example, Campbell and Cochrane (1999), an increase in risk aversion typically leads to a decrease in prices because it increases the price of risk and, consequently, the expected returns as well. In our model, this requires $\xi_1(\gamma)$ to be negative. However, several factors and parameters determine the sign and magnitude of $\xi_1(\gamma)$.

For example, an EIS lower than one and low dividend risk (low d_2) may lead to a positive $\xi_1(\gamma)$. Yet, in our preferred calibration, it is negative and large in absolute value. Thus, the time-varying risk aversion introduces substantial fluctuations in the stock market.

5. The Calibrated Economy with Technology Choice and Time-Varying Risk Aversion

In this section, we present models with and without technology choice and with and without time-varying risk aversion. After presenting the calibration of the models, our discussion is centered on the unique feature of the model with technology choice and time-varying risk aversion. Namely, the cyclical evolution of the conditional volatility of investment and its ability to predict asset pricing moments. Without technology choice or without time-varying risk aversion, the conditional volatility of macroeconomic quantities is constant.

5.1. Calibration

Table 1 shows values for the parameters that are common across models. Specifically, the productivity mean growth rate (μ) is 0.4%, persistence of productivity shocks (ϕ) is 0.9999, the capital share (α) is 0.36, and the quarterly depreciation rate (δ) is 2.1%. When relative risk aversion (γ) is constant, we set it to five and the EIS is 1.5. All these parameter values are taken from Kaltenbrunner and Lochstoer (2010). When risk aversion varies, we set its steady state value to five. Table 1 also shows the exposure of the exogenous market dividends to growth (d_1), exogenous shocks (d_2), and idiosyncratic volatility (d_3), respectively, that are calibrated to the historical moments of aggregate stock market dividends as well as the contemporaneous correlation with consumption growth. These parameter values slightly differ from the ones used in Kaltenbrunner and Lochstoer (2010) as our data cover a longer period.

Table 2 shows values for the parameters that vary across models. TCV is our model with technology choice and time-varying risk aversion. NTCV stands for no technology choice with time-varying risk aversion. TCC denotes an economy with technology choice and constant risk aversion. NTCC is the no technology choice and constant risk aversion benchmark corresponding to model LRR II in Kaltenbrunner and Lochstoer (2010), albeit with slightly different parameters.

For each model, we determine the remaining parameter values by matching moments of the data, which we collect from the NIPA tables, the Bureau of Labor Statistics, CRSP, and WRDS. A detailed description of the data are in Online Appendix D. Specifically, we use the subjective discount factor (β) to match the

mean of the risk-free rate, the capital adjustment cost parameter (χ) to match the ratio of the volatility of consumption growth to the volatility of output growth, the technology choice parameter (ν) to match the volatility of the risk-free rate, and the volatility of exogenous productivity shocks (σ) to match the volatility of consumption growth. Lastly, we set the linear coefficient η_1 to match the volatility of stock market returns and the quadratic coefficient η_2 to bound γ_t away from zero.

5.2. Performance of the Models Vis-à-vis the Data

We evaluate the performance of the models with respect to the data in Table 3. Columns 2 and 3 show the mean estimate of standard macrofinance variables from the data along with their standard errors (*s.e.*), which are Newey and West (1987) corrected with 24 lags. For each model, we report corresponding averages obtained from 1,000 simulated paths with 300 quarters, where we use a burn-in of 100 quarters. The parentheses next to the model statistics show the *t*-statistics (*t* – *st*) of the hypotheses that the data estimates are generated from the model averages.

From Table 3, we see that we cannot reject the hypotheses that the average consumption growth, the volatility of consumption growth, the first autocorrelation of consumption growth, and the ratio of the volatility of consumption growth to the volatility of output growth are generated by the four models. To the contrary, we reject the hypotheses that the ratio of the volatility of investment growth to the volatility of output growth are generated by the four models. In fact, our technology choice model performs slightly worse compared with the standard RBC model in this respect, because investment is negatively autocorrelated at quarterly frequency. This is because technology choice causes investment to react negatively to exogenous shocks, as a way to smooth the effect of negative persistence shocks to productivity. In Online Appendix F, we plot impulse responses where we see that for model TCV, investment decreases upon a positive shock, whereas in a standard RBC economy investment always increases. For the same reason, the annual autocorrelations of consumption, output, and investment are lower compared with the standard RBC model.

All the models replicate the level of the risk-free rate; but only for the two models with technology choice, TCV and TCC, we cannot reject the null hypothesis that volatility of the risk-free rate is generated by either TCV or TCC. Table 3 also shows the expected excess return on investments, the volatility of investment returns, and the Sharpe ratio of investment returns for the models. We see that the model with technology choice and time-varying risk aversion has the lowest quarterly return but also has

Table 1. Common Model Parameters—Quarterly Frequency

Description	Parameter	Value
Exogenous productivity mean growth rate	μ	0.4%
Persistence of exogenous productivity shocks	ϕ	0.9999
Output capital share	α	0.36
Capital depreciation rate	δ	2.1%
(Mean) coefficient of relative risk aversion	γ	5.0
Elasticity of intertemporal substitution	ψ	1.50
Market dividend growth exposure to $\theta_t - c_t$	d_1	0.055
Market dividend growth exposure to exogenous shocks	d_2	0.70
Market dividend growth idiosyncratic volatility	d_3	5.7%

the second lowest volatility of investment returns. Hence, its investment-based Sharpe ratio is not only comparable to the ones of the other models but is second only to the one of the model with technology choice with constant risk aversion. Further, we see that there is no difference between the calibrated aggregate stock market dividends across the models.

Turning to the statistics on the log price-dividend ratio, we see that our model perfectly matches the mean estimate of the log price-dividend ratio of the data without targeting it in the calibration. In addition, our model is the only one where we cannot reject the null hypothesis that the data are generated by the model. The model with technology choice and time-varying risk aversion produces the largest volatility for the log price-dividend ratio. Nevertheless, in this case, the null hypotheses is rejected for all models.¹⁵ In the data, the correlation between the log price-dividend ratio and the risk-free rate is basically zero. It is in general very difficult for asset pricing models to produce such a low correlation. For example, the model of Kaltenbrunner and Lochstoer (2010), without technology choice and constant risk aversion in our calibration, produces a correlation between these two quantities of one. In our model, the correlation between the log price-dividend ratio and the risk-free

rate turns out to be 0.31; only for this model, we cannot reject the null hypothesis.

Why is the correlation between the log price-dividend ratio and the risk-free rate low in our model with technology choice and time-varying risk aversion? It is because in our calibration technology choice and risk aversion move counter to an exogenous shock thereby delaying any impact of the shock on investment and consumption. This produces predictability in consumption growth, which generates fluctuations in the risk-free rate; but because the predictability is short lived, it does not drive the price of a claim on dividends or consumption. Thus, technology choice increases the volatility of the risk-free rate and reduces the correlation of the risk-free rate with the price-dividend ratio.

Next, we discuss the performance of the models on producing realistic returns for the claim to the exogenous dividend stream. In the data, the average quarterly excess return on the aggregate stock market is 2.04. In the models, it ranges from 1.01–1.94. Although the range across the models is large, this difference is not due to technology choice. As pointed out in Kaltenbrunner and Lochstoer (2010), the equity premium in the production-based asset pricing model with long-run risk is quite sensitive to the parameters

Table 2. Calibrated Model Parameters

Description	Parameter	Values			
		TCV	NTCV	TCC	NTCC
Subjective discount factor	β	0.9991	0.9981	0.9991	0.9981
Capital adjustment cost parameter	χ	12.4	1,000	12.5	1,500
Technology choice parameter	ν	6.5	∞	6.0	∞
Volatility of exogenous productivity shocks	σ	4.17%	4.46%	4.21%	4.46%
Linear coefficient of risk aversion	η_1	3.15	2.45		
Quadratic coefficient of risk aversion	η_2	0.482	0.252		
		Averages across simulations			
Mean of relative risk aversion		5.18	5.11		
Standard deviation of relative risk aversion		0.96	0.81		
Minimum of relative risk aversion		3.34	3.55		
Maximum of relative risk aversion		7.25	6.83		

Note. TCV, the model with technology choice and time-varying risk aversion; NTCV, the model without technology choice but with time-varying risk aversion; TCC, an economy with technology choice and constant risk aversion; NTCC, the no technology choice and constant risk aversion benchmark.

Table 3. Calibrated Models vs. Data

	Data		TCV		NTCV		TCC		NTCC	
	<i>est.</i>	<i>s.e.</i>	<i>avg.</i>	<i>t - st</i>	<i>avg.</i>	<i>t - st</i>	<i>avg.</i>	<i>t - st</i>	<i>avg.</i>	<i>t - st</i>
$\mu(\Delta c)$	1.74	(0.38)	1.58	(0.41)	1.58	(0.42)	1.58	(0.41)	1.58	(0.42)
$\sigma(\Delta c)$	2.70	(0.54)	2.70	(0.00)	2.70	(0.00)	2.70	(0.01)	2.70	(0.00)
$ac_1(\Delta c)$	0.48	(0.07)	0.44	(0.58)	0.53	(0.68)	0.46	(0.34)	0.53	(0.69)
$\sigma(\Delta c)/\sigma(\Delta y)$	0.55	(0.06)	0.55	(0.05)	0.55	(0.15)	0.55	(0.05)	0.55	(0.14)
$\sigma(\Delta i)/\sigma(\Delta y)$	2.71	(0.14)	1.86	(6.10)	1.94	(5.53)	1.87	(6.04)	1.94	(5.53)
$ac_1(\Delta y)$	0.53	(0.09)	0.15	(4.36)	0.27	(2.95)	0.16	(4.25)	0.27	(2.95)
$ac_1(\Delta i)$	0.41	(0.15)	0.02	(2.65)	0.18	(1.55)	0.02	(2.68)	0.18	(1.55)
$\mu(R_f)$	0.14	(0.15)	0.14	(0.02)	0.21	(0.44)	0.12	(0.16)	0.22	(0.49)
$\sigma(R_f)$	0.84	(0.10)	0.83	(0.05)	0.27	(5.60)	0.85	(0.09)	0.24	(5.82)
$\mu(R_i - R_f)$			0.29		0.51		0.35		0.37	
$\sigma(R_i)$			1.78		3.07		1.66		2.21	
SR_i			0.21		0.17		0.24		0.17	
$\sigma(\Delta d)$	11.10	(2.12)	11.03	(0.03)	11.21	(0.05)	11.06	(0.02)	11.21	(0.05)
$ac_1(\Delta d)$	0.18	(0.14)	0.27	(0.59)	0.27	(0.64)	0.27	(0.60)	0.27	(0.64)
$\rho(\Delta c, \Delta d)$	0.52	(0.15)	0.49	(0.21)	0.53	(0.11)	0.49	(0.16)	0.53	(0.11)
$\mu(p - d)$	4.79	(0.10)	4.80	(0.12)	5.13	(3.27)	5.03	(2.31)	5.32	(5.11)
$\sigma(p - d)$	0.44	(0.05)	0.33	(2.04)	0.29	(2.89)	0.10	(6.21)	0.10	(6.34)
$\rho(p - d, r_f)$	0.03	(0.17)	0.31	(1.65)	0.85	(4.80)	0.40	(2.19)	1.00	(5.70)
$\mu(R_m - R_f)$	2.04	(0.39)	1.94	(0.25)	1.53	(1.30)	1.24	(2.03)	1.01	(2.61)
$\sigma(R_m)$	11.16	(2.21)	11.14	(0.01)	11.18	(0.01)	7.82	(1.51)	8.32	(1.29)
SR_m	0.18	(0.05)	0.18	(0.05)	0.14	(0.91)	0.16	(0.47)	0.12	(1.23)

Notes. The first-difference of the natural logarithm of a variable X is Δx . The natural logarithm of total output is y ; c denotes total consumption; i denotes total investment. For a variable x , $\sigma(x)$ denotes its volatility; $ac_1(x)$ is its first-order autocorrelation, and $\rho(x, z)$ is its correlation with variable z . The data are described in Online Appendix D. The parentheses next to the data estimates show the standard errors (*s.e.*), which are Newey and West (1987) corrected with 24 lags. The model statistics are averages of 1,000 simulated paths of 300 quarters with a burn-in of 100 quarters. The parentheses next to the model statistics show the t -statistics ($t - st$) of the hypotheses that the data estimates are generated from model averages. Macroeconomic data are annual: 1929–2017. The corresponding data from the models are time aggregated. Price data are quarterly: 1927–2017. TCV, the model with technology choice and time-varying risk aversion; NTCV, the model without technology choice but with time-varying risk aversion; TCC, an economy with technology choice and constant risk aversion; NTCC, the no technology choice and constant risk aversion benchmark.

and especially so to β . Using the parameter values of the TCV model for the benchmark economy (NTCC) leads to an average excess stock market return in the NTCC that is higher than the one in the TCV model. This can be seen from panel (a) of Figure 1, which shows that the equity premium for the TCV model increases with ν .¹⁶ This is also consistent with the relation in (24), which shows that typically the price of risk increases with ν , that is, it increases as technology choice becomes more inflexible. In addition, panel (b) of Figure 1 also shows how the volatility of the risk-free rate varies with ν for the TCV model.

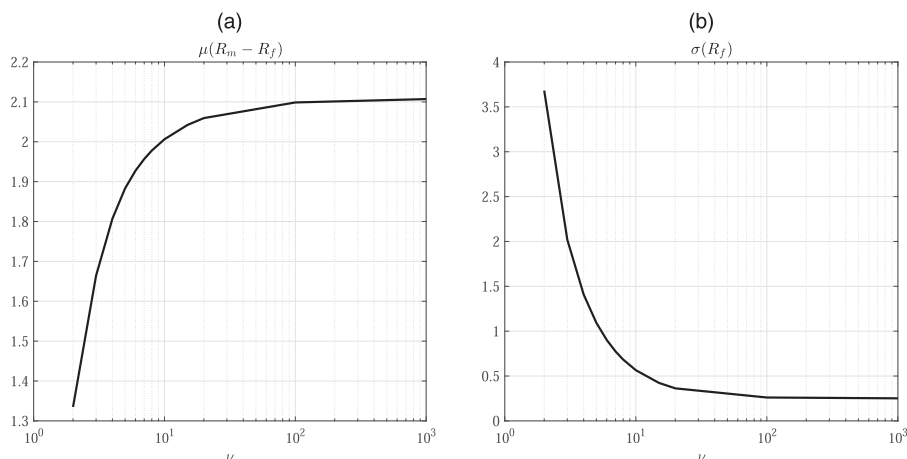
However, the calibration of the model without technology choice and with constant risk aversion that fits the market Sharpe ratio performs less well than shown in Table 3 on the targeted moments of aggregate consumption and the risk-free rate. Hence, overall there is no economically significant difference between the four models on their ability to replicate stock market moments. This point is supported by the models' stock market volatility and Sharpe ratio because we cannot reject the null hypothesis for both

variables in all four cases. In addition, because stock market volatilities in all models are close enough to the data, it is not surprising that there is little variation in risk aversion in the TCV model, which ranges from 3.34–7.25. Nevertheless, the time-varying risk aversion does increase the volatility from roughly 8% per quarter to 11.14% (TCV) or 11.18% (NTCV) per quarter, which perfectly matches the data.

We have also calibrated four economies with low EIS. The calibrated parameter values and the performance of those models vis-à-vis the data are summarized in Tables 7 and 8 in Online Appendix E. Overall, the low EIS cases perform about equally well with respect to the empirical moments shown in Table 3, although they require quite high levels of risk aversion and subjective discount factor.

In Table 4, we show predictive regression of excess stock market returns by the log price-dividend ratio. We see that only the models with time-varying risk aversion generate a realistic level for the standardized regression coefficient. Specifically, in the data, the absolute level of the coefficient increases from 0.13

Figure 1. $\mu(R_m - R_f)$ and $\sigma(R_f)$ of the TCV Model with High EIS



Note. The plots show how the expected excess market return (panel (a)) and the volatility of the risk-free rate (panel (b)) vary with the technology choice parameter ν for the TCV.

at 1 quarter to 0.53 at 28 quarters with t -statistics ranging from 3.03–3.75. The TCV and NTCV models generate about half of the predictability in the data and this for all horizons. Further, we cannot reject the null hypothesis that the data are generated by either the TCV or NTCV model. In both models with constant risk aversion, the regression coefficients are about one-third of the ones in the models with time-varying risk aversion and the null is rejected for every single regression coefficient. From Table 9 in Online Appendix E, we learn that these results reproduce also when we use a low EIS.

In summary, the model without technology choice has difficulty simultaneously matching the targeted moments discussed above along with the volatility of the risk-free rate, the moments of the log price-dividend ratio, and the correlation between the risk-free rate and the log price-dividend ratio. In particular, only the model with technology choice and time-varying risk aversion reproduces the correlation between the risk-free rate and the log price-dividend ratio. In addition, the model without time-varying risk aversion has difficulty in producing predictability in excess returns by the log price-dividend ratio.

Table 4. Excess Return Predictability by $p - d$

	$\rho(p - d_t, \sum_{s=1}^{\tau} [R_{m,t+s} - R_{f,t+s-1}])$								
	$\tau = 1$	$\tau = 2$	$\tau = 4$	$\tau = 8$	$\tau = 12$	$\tau = 16$	$\tau = 20$	$\tau = 24$	$\tau = 28$
Data	-0.13	-0.19	-0.26	-0.35	-0.39	-0.42	-0.47	-0.50	-0.53
$t - st$	(3.03)	(3.33)	(3.34)	(3.23)	(2.97)	(2.81)	(3.20)	(3.52)	(3.75)
TCV	-0.07	-0.10	-0.14	-0.19	-0.23	-0.26	-0.29	-0.31	-0.33
$t - st$	(1.42)	(1.59)	(1.57)	(1.44)	(1.23)	(1.07)	(1.22)	(1.33)	(1.39)
NTCV	-0.07	-0.10	-0.14	-0.20	-0.24	-0.27	-0.30	-0.32	-0.34
$t - st$	(1.35)	(1.52)	(1.50)	(1.37)	(1.17)	(1.00)	(1.15)	(1.26)	(1.31)
TCC	-0.02	-0.03	-0.05	-0.06	-0.08	-0.09	-0.10	-0.11	-0.11
$t - st$	(2.48)	(2.74)	(2.75)	(2.63)	(2.38)	(2.22)	(2.53)	(2.76)	(2.93)
NTCC	-0.02	-0.03	-0.05	-0.06	-0.08	-0.09	-0.10	-0.11	-0.11
$t - st$	(2.47)	(2.73)	(2.73)	(2.62)	(2.38)	(2.22)	(2.52)	(2.76)	(2.93)

Notes. The table shows the standardized regression coefficient on the log price-dividend ratio from a standard predictive regression using the stock market excess return for horizons from 1 quarter to 28 quarters. The data are described in Online Appendix D. The t -statistics ($t - st$) for the data are for the null hypothesis that the regression coefficients are zero. Standard errors are Newey and West (1987) corrected with 24 lags. The model statistics are averages of 1,000 simulated paths of 300 quarters with a burn-in of 100 quarters. The parentheses below the regression coefficients for the models show the t -statistics ($t - st$) of the hypotheses that the data estimates are generated from model averages. The data are quarterly: 1947–2017. TCV, the model with technology choice and time-varying risk aversion; NTCV, the model without technology choice but with time-varying risk aversion; TCC, an economy with technology choice and constant risk aversion; NTCC, the no technology choice and constant risk-aversion benchmark.

5.3. Testing the Model

After establishing that the model with technology choice and time-varying risk aversion reproduces standard macroeconomic and asset pricing moments at least as well as a benchmark model à la Kaltenbrunner and Lochstoer (2010), we now turn to empirical evidence on the unique features of our model.

Here, we explore the ability of the model with technology choice and time-varying risk aversion to replicate the conditional volatility of output, consumption, and investment. In the other models, the conditional volatilities are constant because we obtain time-varying conditional volatilities only if σ_ω is time varying. We do this for the high and low EIS cases.

From Table 5, we see that the technology choice model with time-varying risk aversion and high EIS does a fairly good job in replicating the fluctuations in the conditional volatilities of output, consumption, and investment. Specifically, in the data these standardized volatilities are 0.32, 0.43, and 0.37 in the constrained estimation, whereas in our model with high EIS, these are 0.39, 0.42, and 0.35, respectively.¹⁷ The conditional volatilities behave similarly in the unconstrained estimation, and we cannot reject the null hypotheses for consumption and investment, whereas for output the fluctuations generated by the model are slightly larger compared with the data. For the model with low EIS, all null hypotheses are rejected. In addition, the first-order autocorrelation of these three macroeconomic volatilities are very close to the data. Only for the unconstrained

autocorrelation of the volatility of output do we reject the null. For the model with low EIS, only the null hypotheses for the autocorrelation of the volatility consumption are not rejected.

We emphasize the correlations between the log price-dividend ratio and the conditional volatilities of output, consumption, and investment in Table 5. Although these results are mixed, they have important implication for a relation between current macroeconomic quantities and expected excess returns. First, we see that in the data the correlation between the log price-dividend ratio and the volatility of investment is either -0.65 or -0.67 , depending on whether we constrain the estimation or not. Second, in the technology choice model with time-varying risk aversion and high EIS, the average is -0.64 . For the economy with low EIS, the average is 0.61 . From this, we expect that in predictive regressions with the conditional volatility of investment the high EIS model produces a sign that is in line with the data, whereas the model with low EIS produces the wrong sign for the regression coefficient. Further, although the model with high EIS does fairly well on the correlation between the log price-dividend ratio and the volatility of output, its average is a bit lower than in the data. Consequently, the consumption volatility is not only lower than what we see in the data but also slightly positive instead of negative.

Finally, we discuss the predictive regression analysis in Table 6. We start by establishing that the conditional volatility of investment predicts excess stock

Table 5. Conditional Volatilities

	Data				High EIS			Low EIS		
	Constrained		Unconstrained		avg.	$t - st$	$t - st$	avg.	$t - st$	$t - st$
	est.	s.e.	est.	s.e.		(con.)	(unc.)		(con.)	(unc.)
$\sigma(\sigma_y)/\mu(\sigma_y)$	0.32	(0.04)	0.32	(0.02)	0.39	(1.96)	(3.48)	0.47	(4.08)	(7.04)
$\sigma(\sigma_c)/\mu(\sigma_c)$	0.43	(0.06)	0.35	(0.04)	0.42	(0.20)	(1.60)	0.17	(4.53)	(4.41)
$\sigma(\sigma_i)/\mu(\sigma_i)$	0.37	(0.06)	0.30	(0.05)	0.35	(0.43)	(0.87)	0.59	(3.79)	(5.42)
$ac(\sigma_y)$	0.98	(0.01)	0.89	(0.02)	0.97	(1.23)	(3.19)	0.94	(4.14)	(2.00)
$ac(\sigma_c)$	0.97	(0.02)	0.93	(0.02)	0.97	(0.33)	(1.64)	0.95	(1.39)	(0.87)
$ac(\sigma_i)$	0.98	(0.02)	0.98	(0.02)	0.97	(0.82)	(0.34)	0.92	(4.10)	(3.33)
$\rho(p - d, \sigma_y)$	-0.75	(0.07)	-0.59	(0.07)	-0.46	(4.23)	(1.77)	0.88	(23.37)	(20.01)
$\rho(p - d, \sigma_c)$	-0.82	(0.04)	-0.76	(0.05)	0.31	(27.31)	(20.44)	0.94	(42.35)	(32.28)
$\rho(p - d, \sigma_i)$	-0.65	(0.08)	-0.67	(0.08)	-0.64	(0.12)	(0.29)	0.61	(16.00)	(16.86)

Notes. For a variable x , $\sigma(\sigma_x)/\mu(\sigma_x)$ denotes its volatility of volatility normalized by the mean of volatility; $ac_1(\sigma_x)$ is its first-order autocorrelation, and $\rho(\sigma_x, z)$ is its correlation with variable z . The data are described in Online Appendix D. The parentheses next to the data estimates show the standard errors (*s.e.*), which are Newey and West (1987) corrected with 24 lags. The conditional volatility series of output, consumption, and investment in the data are obtained by fitting an ARMA(1,1)-EGARCH(1,1) to each growth rate series. In the constrained estimation for the process for volatility (EGARCH), we set the AR(1) coefficient equal to 0.9999 to mimic the persistence of the exogenous shocks in the model economies. We use *con.* and *unc.* as shorthand for constrained and unconstrained, respectively. Both the high and low EIS economies correspond to the TCV model, which is the model with technology choice and time-varying risk aversion. The model statistics are averages of 1,000 simulated paths of 300 quarters with a burn-in of 100 quarters. The parentheses below the regression coefficients for the models show the t -statistics ($t - st$) of the hypotheses that the data estimates are generated from model averages. The data are quarterly: 1947–2017.

Table 6. Excess Return Predictability by Conditional Investment Volatilities

	$\sigma_i: \rho(\sigma_{i,t}, \sum_{s=1}^{\tau} [R_{m,t+s} - R_{f,t+s-1}])$								
	$\tau = 1$	$\tau = 2$	$\tau = 4$	$\tau = 8$	$\tau = 12$	$\tau = 16$	$\tau = 20$	$\tau = 24$	$\tau = 28$
Data (con.)	0.11	0.15	0.21	0.31	0.39	0.43	0.45	0.49	0.48
$t - st$	(2.53)	(2.47)	(2.53)	(2.71)	(2.94)	(3.01)	(3.30)	(3.57)	(3.71)
Data (unc.)	0.12	0.15	0.21	0.32	0.41	0.45	0.48	0.52	0.52
$t - st$	(2.61)	(2.51)	(2.57)	(2.76)	(3.04)	(3.14)	(3.50)	(3.86)	(4.06)
High EIS	0.04	0.06	0.08	0.12	0.14	0.17	0.18	0.20	0.21
$t - st$ (con.)	(1.58)	(1.47)	(1.51)	(1.68)	(1.85)	(1.86)	(1.95)	(2.11)	(2.05)
$t - st$ (unc.)	(1.65)	(1.50)	(1.55)	(1.74)	(1.97)	(2.01)	(2.15)	(2.38)	(2.37)
Low EIS	-0.05	-0.07	-0.10	-0.14	-0.17	-0.18	-0.20	-0.21	-0.22
$t - st$ (con.)	(3.74)	(3.73)	(3.80)	(3.94)	(4.19)	(4.30)	(4.75)	(5.11)	(5.40)
$t - st$ (unc.)	(3.83)	(3.77)	(3.82)	(3.97)	(4.27)	(4.41)	(4.96)	(5.40)	(5.78)

Notes. The table shows the standardized regression coefficient on the conditional volatility of investment from a standard predictive regression using the stock market excess return for horizons from 1 quarter to 28 quarters. The data are described in Online Appendix D. The t -statistics ($t - st$) for the data are for the null hypothesis that the regression coefficients are zero. Standard errors are Newey and West (1987) corrected with 24 lags. The conditional volatility series of output, consumption, and investment in the data are obtained by fitting an ARMA(1,1)-EGARCH(1,1) to each growth rate series. In the constrained estimation for the process for volatility (EGARCH), we set the AR(1) coefficient equal to 0.9999 to mimic the persistence of the exogenous shocks in the model economies. We use con. and unc. as shorthand for constrained and unconstrained, respectively. Both the high and low EIS economies correspond to the TCV model, which is the model with technology choice and time-varying risk aversion. The model statistics are averages of 1,000 simulated paths of 300 quarters with a burn-in of 100 quarters. The parentheses below the regression coefficients for the models show the t -statistics ($t - st$) of the hypotheses that the data estimates are generated from model averages. The data are quarterly: 1947–2017.

market returns at least as well as the log price-dividend ratio. The standardized regression coefficients at both the short horizon of 1 quarter and the long horizon of 24 quarters are basically identical to the ones using the log price-dividend ratio, and this is independent of whether the estimation is constrained or unconstrained. The t -statistics ($t - st$), which are Newey and West (1987) corrected with 24 lags, range from 2.5–4.0. Turning to our technology choice model with time-varying risk aversion and high EIS, we see that the model produces about half of the predictability at all horizons, which is consistent with the evidence in Table 4. Here, however, the slope of the standardized regression coefficients over the regression horizon is not steep enough; thus, we reject the null hypotheses from 20 (12) quarters on for the constrained (unconstrained) estimation. Even then, Table 6 strongly supports our model and rules out a specification with low EIS.

We close by referring to Table 10 in the Online Appendix, where the conditional volatility series of investment are filtered out of the simulated investment data. From this exercise, we see that an empiricist cannot accidentally detect predictability in the models without technology choice or without time-varying risk aversion, where the true investment volatility is constant.

5.4. Discussing the Macroeconomy Link

The fact that we cannot observe how TFP is determined begs the question whether the standard RBC

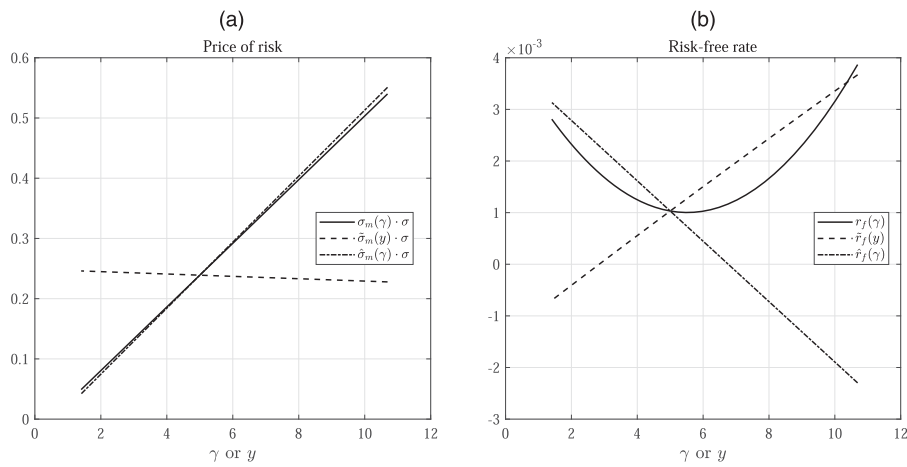
model can be amended with independent shocks to the conditional volatility of TFP to reproduce the results of our model. Briefly, this would generate fluctuations in the conditional volatility of all the macroeconomic quantities but would not produce predictability of returns by the conditional volatility of investment.

To elaborate, we examine how the price of risk depends on the conditional volatility of TFP. Consider the price of risk from Equation (22b),

$$\sigma_m = \frac{1}{\psi} \sigma_c(y) + \left(\gamma - \frac{1}{\psi} \right) \sigma_u(y),$$

where the conditional volatility of TFP $\sigma_\omega(y)$ is driven by some variable y . We then consider three cases: (i) TCV where y is risk aversion γ , (ii) where we vary only y and keep $\gamma = 5$, and (iii) where we vary γ and let $y = 5$. Panel (a) of Figure 2 plots the price of risk for these three cases as functions of γ or y . We see that the price of risk for TCV (solid line) depends positively on γ , whereas the price of risk in the second case (dashed line, $\tilde{\sigma}_m$) is almost flat and slightly decreasing in y . Further, the price of risk for the third case (dashed-dotted line, $\hat{\sigma}_m$) is almost identical to that of TCV. From these results, it is clear that, in our model, the conditional volatility of TFP has a negligible effect on the price of risk. Thus, in a standard RBC model with independent shocks to σ_ω , the conditional volatility of investment cannot predict stock returns.

Figure 2. Price of Risk for TCV



Notes. Panel (a) shows the price of risk as a function of risk aversion γ or y in three alternative model specifications: $\sigma_m(\gamma)$ corresponds to model TCV; $\hat{\sigma}_m(y) = \frac{1}{\psi} \sigma_c(y) + (\bar{\gamma} - \frac{1}{\psi}) \sigma_u(y)$ corresponds to the case where conditional volatilities vary as in TCV but depend on some variable y ; and $\hat{\sigma}_m(\gamma) = \frac{1}{\psi} \sigma_c(\bar{\gamma}) + (\gamma - \frac{1}{\psi}) \sigma_u(\bar{\gamma})$ corresponds to the case where the conditional volatilities are constant. Panel (b) shows the risk-free rate as a function of risk aversion γ or y in the same three alternative model specifications, assuming that θ and k take their steady state values: $r_f(\gamma)$ corresponds to model TCV; $\hat{r}_f(y)$ corresponds to the case where the conditional volatilities vary as in TCV but driven by some variable y ; and $\hat{r}_f(\gamma)$ corresponds to the case where the conditional volatilities are constant. In the above, we set $\bar{\gamma} = 5$.

Technology choice also generates time-varying means in all macroeconomic quantities, which helps to reproduce the fluctuations in the risk-free rate. For instance, the time-varying mean breaks the perfect correlation between the risk-free rate and the price-dividend ratio. Again, the time-variation in risk aversion matters as it further reduces this correlation by introducing non-monotonicity in the risk-free rate, as can be seen from panel (b) of Figure 2 (solid line).

To summarize, the standard RBC model can be modified to reproduce all our results if the conditions of Corollary 3 are satisfied. Specifically, this requires time variation in the mean and the volatility of TFP and that risk aversion drives the volatility of TFP.

6. Conclusions

In this paper, we embark on an abstract exploration of technology choice or state-contingent technology in a production-based economy. Our point of departure is that it is plausible to assume that production technology is state dependent. Following the literature on consumption-based asset pricing, we also assume that risk aversion is state dependent.

Although technology choice directly depends on risk aversion, it remains that risk aversion does not directly affect the macroeconomy, but only through technology choice. Specifically, in our model with technology choice, we see that if risk aversion is time varying, then the conditional volatility of investment evolves with risk aversion. We also see that the parameter that governs technology choice, and through that the cost of productivity transformation, also governs the volatility of the risk-free rate.

In our preferred calibration, technology choice and risk aversion move counter to an exogenous shock. Thereby, they delay the reaction of investment and consumption to the shock. Therefore, we see predictability in consumption growth, which generates fluctuations in the risk-free rate. Because technology choice is one-period ahead, the generated predictability is short lived. It, thus, does not affect long-lived securities, such as the claim to aggregate dividends. In the model, we see that the volatility of the risk-free rate increases, but the volatility of the log price-dividend ratio does not. This mechanism, hence, reduces the correlation of the risk-free rate with the log price-dividend ratio. Because in the data there almost is no correlation between the risk-free rate and the log price-dividend ratio and because without the mechanism in our model it is difficult to significantly reduce the correlation below one, we think that this is a useful way to think about the impact of technology choice.

To further strengthen our point that asset prices and the macroeconomy are linked through variations in risk aversion, we regress excess stock market returns on the conditional volatility of investment growth and show that the model reproduces about half of the predictability in the data. This novel empirical evidence reproduces only in a model with technology choice and time-varying risk aversion.

We close by reiterating that it would be desirable to provide microfoundations for the stylized production technology employed in the model. We leave this ambitious task for future research.

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Endnotes

¹ Cochrane (2008) calls this defect of standard real business cycle models the divorce between asset pricing and macroeconomics.

² See, for example, Campbell and Cochrane (1999), Chan and Kogan (2002), Xiouros and Zapatero (2010), and Ehling and Heyerdahl-Larsen (2017).

³ A practicable way to substitute productivity across states is through investing in different production technologies. In Online Appendix A, we provide a theoretical connection between the reduced-form approach to technology choice that we adopt and investing in several technologies as in Jermann (2010). For example, it seems plausible that the different technologies of generating electricity, for example, coal, natural gas, nuclear, oil, solar, wind, etc., are broadly consistent with Jermann (2010) and, therefore, also with our reduced-form approach. Specifically, because each technology has its own risk characteristics, combining them allows choosing the risk profile of energy generation.

⁴ Another mechanism with control over output is the variable capital utilization rate considered by Burnside and Eichenbaum (1996), where the utilization rate is chosen after the realization of the exogenous shock.

⁵ In a contemporaneous contribution, Bretscher et al. (2018) show that endogenous macroeconomic responses to uncertainty shocks are amplified through higher level of risk aversion, by taking into account higher orders in the perturbation method.

⁶ By now, the literature on investment- or production-based asset pricing is vast. Recent contributions include Papanikolaou (2011), Gârleanu et al. (2012), Ai et al. (2013), Belo et al. (2014), Croce (2014), Kung (2015), Kung and Schmid (2015), and Chen (2016) among many others; none of these works, however, study state-contingent technology.

⁷ The relative risk aversion γ_t is not included as a state variable because it is only a function of θ_t .

⁸ See Melino and Yang (2003) and Dew-Becker (2014) for a stochastic discount factor with recursive preferences and time-varying risk aversion.

⁹ For example, a 10% increase in productivity when θ is high has the same cost as a 10% increase in productivity when θ is low. Therefore, increasing productivity when θ is high and decreasing it when it is low maximizes average productivity.

¹⁰ The conditional volatility of TFP, $\sigma_\omega(\gamma_t)$, is independent of the state variables k and θ because the cost of productivity transformation in (4) only depends on the percentage deviation from the natural level of productivity.

¹¹ Log-linearizing the stochastic discount factor (12) gives the following:

$$\ln M_{t,t+1} = -\frac{1}{\psi}(c_{t+1} - c_t) - \left(\gamma_t - \frac{1}{\psi}\right)[u_{t+1} - \mathbb{E}_t(u_{t+1})].$$

¹² This corresponds to the case of utility smoothing discussed in Backus et al. (2013).

¹³ We cannot rule out that c_θ is negative; but for the parameters used in the calibration, we obtain a positive c_θ .

¹⁴ The log-linear approximation of the utility is given as follows:

$$u_t = c_t + \sum_{\tau=1}^{\infty} \hat{\beta}^\tau (\mu_{t+\tau} - \mu),$$

where $\hat{\beta} = \beta e^{\mu(1-1/\psi)}$ and $\mu_{t+\tau} = \mathbb{E}_t(c_{t+\tau+1} - c_t)$.

¹⁵ Although the null is rejected for the TCV model, its volatility of the log price-dividend ratio is 0.33, whereas the NTCC model produces a volatility of 0.10. In addition, Boudoukh et al. (2007) show that in their sample the volatility of the log price-dividend ratio declines from 0.41 to roughly 0.30 with share repurchases, which is closer to what our TCV model generates.

¹⁶ The average excess return in the benchmark economy (NTCC) with the same parameters as in the preferred model (TCV) corresponds to the case where ν tends to infinity.

¹⁷ In the constrained estimation for the process for volatility (EGARCH), we set the AR(1) coefficient equal to 0.9999 to mimic the persistence of the exogenous shocks in the model economies.

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